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POLITECHNIKA POZNAŃSKA  
INSTYTUT ANALIZY KONSTRUKCJI  
ZAKŁAD MECHANIKI BUDOWLI

5,0

ĆWICZENIE NR 1

OBLICZANIE PRZEMIESZCZEŃ Z ZASTOSOWANIEM  
RÓWNAŃ PRACY WIRTUALNEJ

Nazwisko i imię studenta Zofia Rybczyńska  
Rok akademicki 2023/2024  
Semestr III  
Grupa 8 projekty

Data	Uwagi sprawdzającego	Podpis

Dla ramy nr 6 należy:

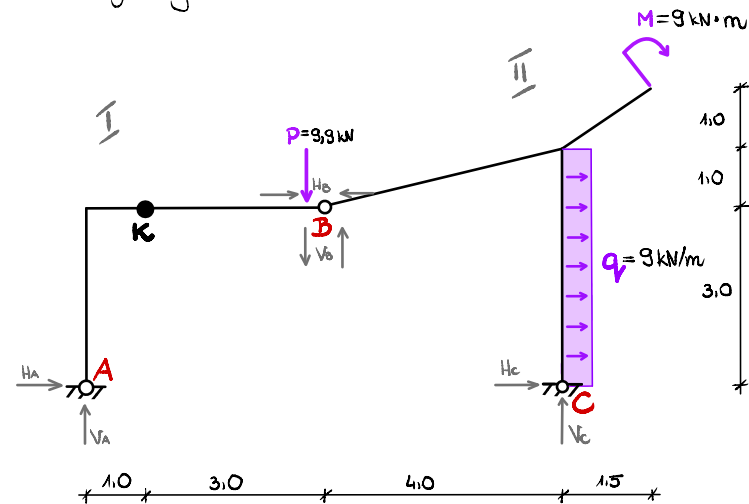
- Przyjąć wstępnie przekroje prętów z profili dwuteowych (IN, IPE, HEB, HEA), tak aby pod działaniem podanego obciążenia powstałe w prętach naprężenia normalne spełniały warunek  $\sigma \leq 200 \text{ MPa}$ . UWAGA: Przyjąć ten sam profil dla wszystkich prętów.
- Obliczyć przemieszczenia zaznaczone w tablicy.

Dla kratownicy nr 2 należy:

- Przyjąć wstępnie przekroje prętów z kształtowników zamkniętych kwadratowych, tak aby pod działaniem podanego obciążenia powstałe w prętach naprężenia normalne spełniały warunek  $\sigma \leq 200 \text{ MPa}$ . UWAGA: Przyjąć ten sam profil dla wszystkich prętów.
- Obliczyć przemieszczenie pionowe/poziome węzła A oraz obrót pręta s. Do obliczeń przyjąć  $E = 205 \text{ GPa}$ .

Przyczyna przemieszczenia	Przem. punktu K składowa pozioma	Przem. punktu K składowa pionowa	Przem. punktu K wypadkowa	Obrót przekroju K
Obciążenie zewnętrzne (wpływ M)	✓	✓	✓	✓
Zmiana temperatury	✓	✓		
Osiadania podpór	✓	✓		

Rybczyńska Zofia



$$P = 1,1 qL$$

$$M = qL^2$$

$$L = 1,0 \text{ m}$$

$$q = 9 \text{ kN/m}$$

$$\sum M_A = 0 \quad (V_c)$$

$$-8V_c + 4 \cdot 9,9 + 9 + 9 \cdot 4 \cdot 2 = 0$$

$$8V_c = 120,6 \quad | :8$$

$$V_c = 15,075 \text{ kN}$$

$$\sum M_C = 0 \quad (V_A)$$

$$8V_A - 4 \cdot 9,9 + 9 + 9 \cdot 4 \cdot 2 = 0$$

$$8V_A = -41,4 \quad | :8$$

$$V_A = -5,175 \text{ kN}$$

$$\text{spr. } \sum Y = 0 \quad (V_A, V_c)$$

$$V_A + V_c - 9,9 = 0$$

$$0 = 0$$

$$\sum M_B = 0 \quad (H_A)$$

$$4V_A - 3H_A = 0$$

$$3H_A = 4V_A \quad |$$

$$3H_A = 4 \cdot (-5,175)$$

$$3H_A = -20,7 \quad | :3$$

$$H_A = -6,9 \text{ kN}$$



$$\sum M_B \text{ II} = 0$$

$$-4V_c - 3H_c - 9 \cdot 4 \cdot 1 + 9 = 0$$

$$3H_c = -4V_c - 9 \cdot 4 \cdot 1 + 9$$

$$3H_c = -87,3$$

$$H_c = -29,1 \text{ kN}$$

$$\sum X = 0 \quad (H_c, H_A)$$

$$H_A + H_c + 9 \cdot 4 = 0$$

$$0 = 0$$

$$\sum X \text{ I} = 0$$

$$-6,9 + H_B = 0$$

$$H_B = 6,9 \text{ kN}$$

$$\sum X \text{ II} = 0$$

$$-H_B - 29,1 + 9 \cdot 4 = 0$$

$$H_B = 6,9 \text{ kN}$$

$$\sum Y \text{ I} = 0$$

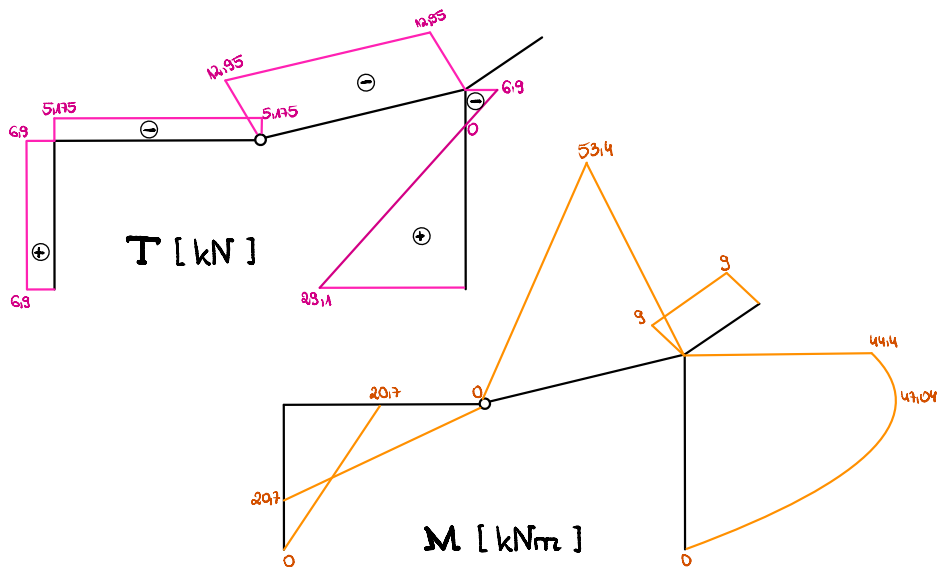
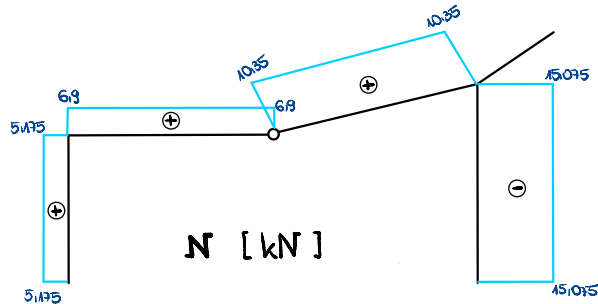
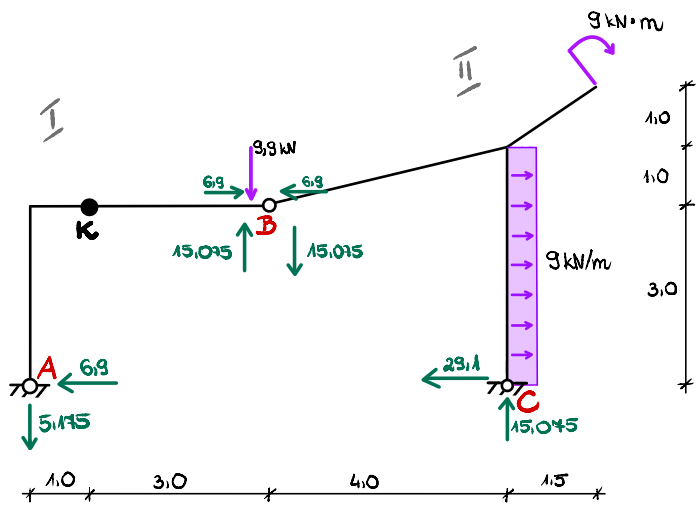
$$-5,175 - V_B - 9,9 = 0$$

$$V_B = -15,075 \text{ kN}$$

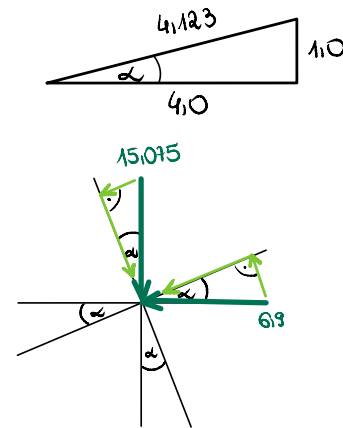
$$\sum Y \text{ II} = 0$$

$$V_B + 15,075 = 0$$

$$V_B = -15,075 \text{ kN}$$



STRONA 3



$$\sqrt{1^2 + 4^2} = \sqrt{17} = 4,123$$

$$\sin \alpha = \frac{1}{4,123} = 0,2425$$

$$\cos \alpha = \frac{4}{4,123} = 0,9702$$

$$N = \oplus 15,075 \cdot \sin \alpha + 6,9 \cdot \cos \alpha = 10,35 \text{ kN}$$

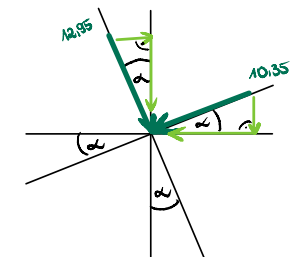
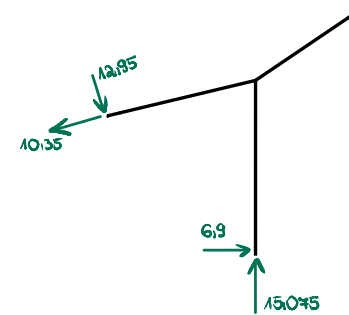
$$T = 15,075 \cdot \cos \alpha - 6,9 \cdot \sin \alpha = 12,95 \text{ kN} \ominus$$

$$29,1 - 9x = 0$$

$$x = \frac{29,1}{9} = 3,233$$

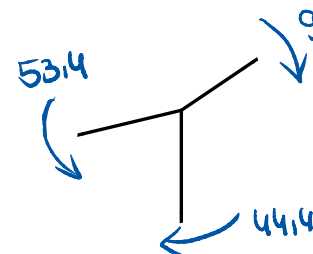
$$29,1 \cdot 3,233 - 9 \cdot 3,233 \cdot \frac{3,233}{2} = 47,04 \text{ kN}\cdot\text{m}$$

$$29,1 \cdot 4 - 9 \cdot 4 \cdot \frac{4}{2} = 44,4 \text{ kN}\cdot\text{m}$$



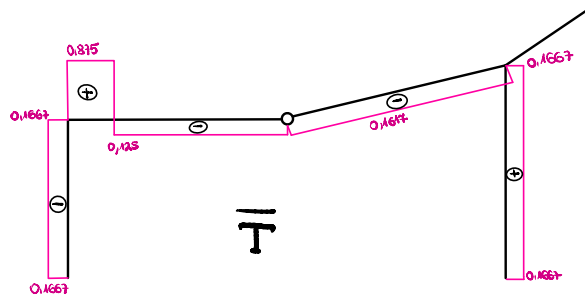
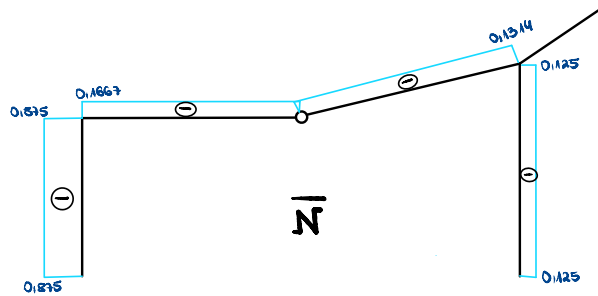
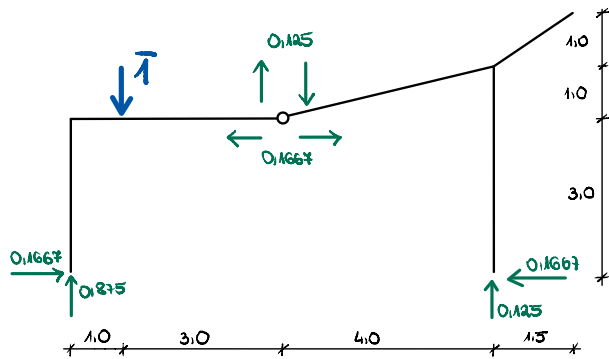
$$15,075 - 12,95 \cdot \cos \alpha - 10,35 \cdot \sin \alpha = 0,001$$

$$6,9 + 12,95 \cdot \sin \alpha - 10,35 \cdot \cos \alpha = -0,0003$$

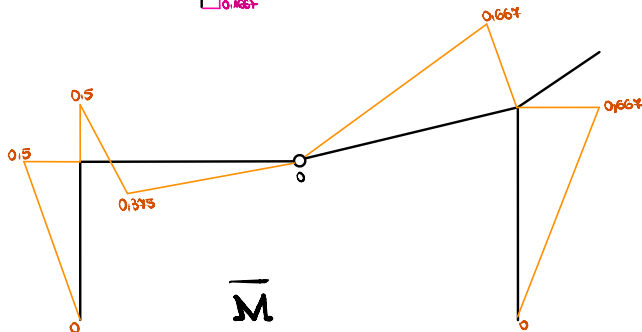


STRONA 4

# 1 PIONOWA

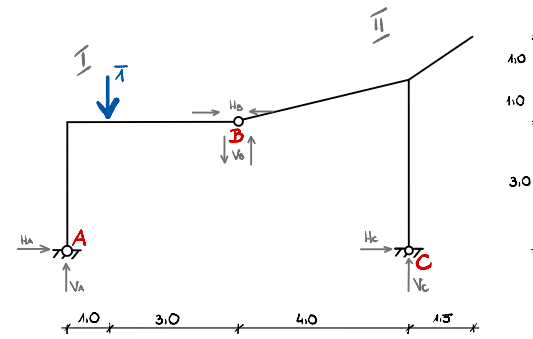


II



M

STRONA 5



$$\begin{aligned} \sum M_A = 0 \quad (V_C) \\ -8V_C + 1 = 0 \\ 8V_C = 1 \quad | :8 \\ V_C = 0,125 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum M_C = 0 \quad (V_A) \\ 8V_A - 1 \cdot 7 = 0 \\ 8V_A = 7 \quad | :8 \\ V_A = 0,875 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{spr. } \sum Y = 0 \quad (V_A, V_C) \\ V_A + V_C - 1 = 0 \\ 0 = 0 \end{aligned}$$

$$\begin{aligned} \sum M_B = 0 \quad (H_A) \\ 4V_A - 3H_A - 1 \cdot 3 = 0 \\ 3H_A = 4 \cdot 0,875 - 1 \cdot 3 \\ 3H_A = 0,5 \quad | :3 \\ H_A = 0,1667 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum M_B = 0 \\ -4H_C - 3H_C = 0 \\ 3H_C = -4 \cdot 0,125 \\ 3H_C = -0,5 \quad | :3 \\ H_C = -0,1667 \text{ kN} \end{aligned}$$

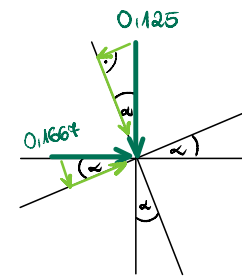
$$\begin{aligned} \sum X = 0 \quad (H_C, H_A) \\ H_A + H_C = 0 \\ 0 = 0 \end{aligned}$$

$$\begin{aligned} \sum X_I = 0 \\ H_A + H_B = 0 \\ H_B = -0,1667 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum X_{II} = 0 \\ -H_B + H_C = 0 \\ H_B = -0,1667 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum Y_I = 0 \\ V_A - 1 - V_B = 0 \\ 0,875 - 1 = V_B \\ V_B = -0,125 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum Y_{II} = 0 \\ V_B + V_C = 0 \\ V_B = -0,125 \text{ kN} \end{aligned}$$

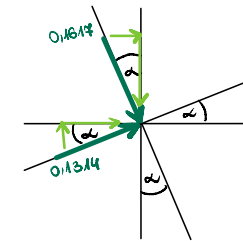


$$\sin \alpha = 0,2425$$

$$\cos \alpha = 0,9702$$

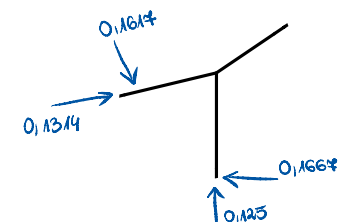
$$N = 0,125 \sin \alpha - 0,1667 \cos \alpha = \ominus 0,1314 \text{ kN}$$

$$T = 0,125 \cos \alpha + 0,1667 \sin \alpha = \ominus 0,1619 \text{ kN}$$



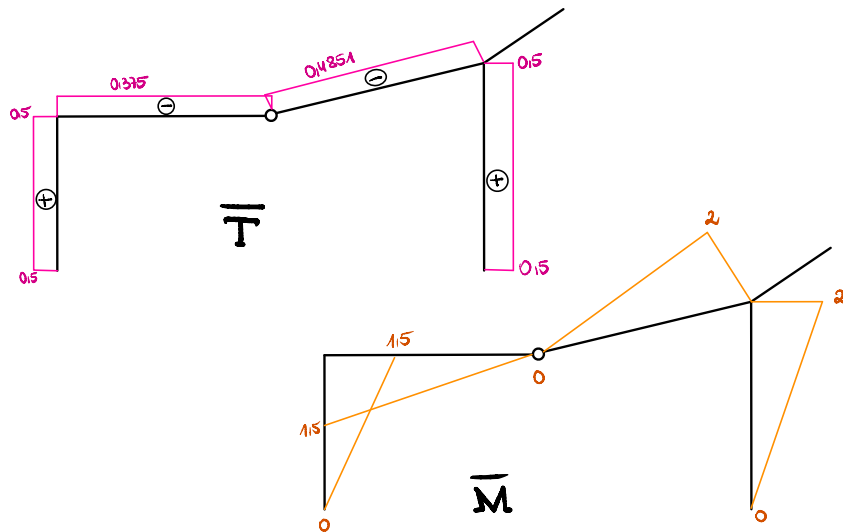
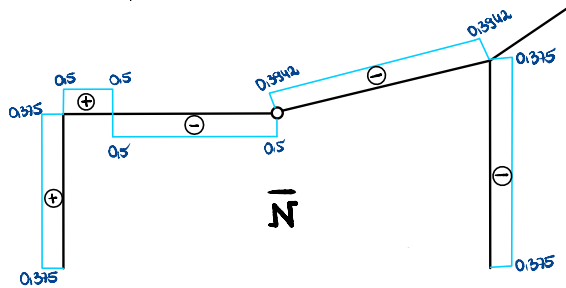
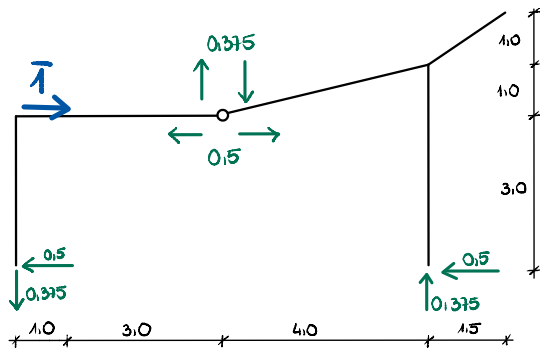
$$0,1619 \sin \alpha + 0,1314 \cos \alpha - 0,1667 = -0,000003$$

$$0,125 + 0,1314 \sin \alpha - 0,1619 \cos \alpha = -0,000002$$

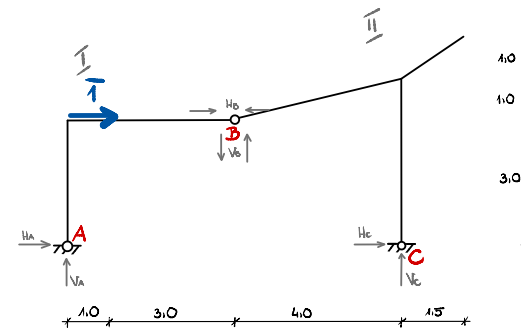


STRONA 6

# 1 POZIOMA



STRONA 7



$$\begin{aligned} \sum M_B \text{ II} &= 0 \\ -4V_C - 3H_C &= 0 \\ -4 \cdot 0,375 &= 3H_C \\ -1,5 &= 3H_C \quad | :3 \\ H_C &= -0,5 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum X &= 0 \quad (H_C, H_A) \\ H_A + H_C + 1 &= 0 \\ 0 &= 0 \end{aligned}$$

$$\begin{aligned} \sum X \text{ I} &= 0 \\ H_A + H_B + 1 &= 0 \\ -0,5 + 1 &= H_B \\ H_B &= -0,5 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum X \text{ II} &= 0 \\ -H_B + H_C &= 0 \\ H_B &= -0,5 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum Y \text{ I} &= 0 \\ V_A - V_B &= 0 \\ V_B &= -0,375 \text{ kN} \end{aligned}$$

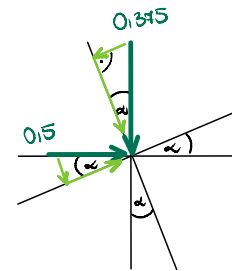
$$\begin{aligned} \sum Y \text{ II} &= 0 \\ V_B + V_C &= 0 \\ V_B &= -0,375 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum M_A &= 0 \quad (V_C) \\ -8V_C + 3 \cdot 1 &= 0 \\ 8V_C &= 3 \quad | :8 \\ V_C &= 0,375 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum M_C &= 0 \quad (V_A) \\ 8V_A + 3 \cdot 1 &= 0 \\ 8V_A &= -3 \quad | :8 \\ V_A &= -0,375 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{Spr. } \sum Y &= 0 \quad (V_A, V_C) \\ V_A + V_C &= 0 \\ 0 &= 0 \end{aligned}$$

$$\begin{aligned} \sum M_B \text{ I} &= 0 \quad (H_A) \\ 4V_A - 3H_A &= 0 \\ 4 \cdot (-0,375) &= 3H_A \\ 3H_A &= -1,5 \quad | :3 \\ H_A &= -0,5 \text{ kN} \end{aligned}$$

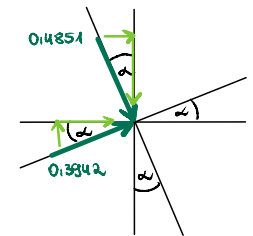


$$\sin \alpha = 0,2425$$

$$\cos \alpha = 0,9702$$

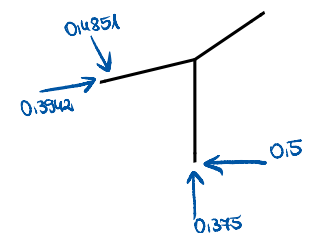
$$N = 0,5 \cdot \cos \alpha - 0,375 \cdot \sin \alpha = 0,3942 \text{ kN}$$

$$T = 0,5 \cdot \sin \alpha + 0,375 \cdot \cos \alpha = 0,4851 \text{ kN}$$

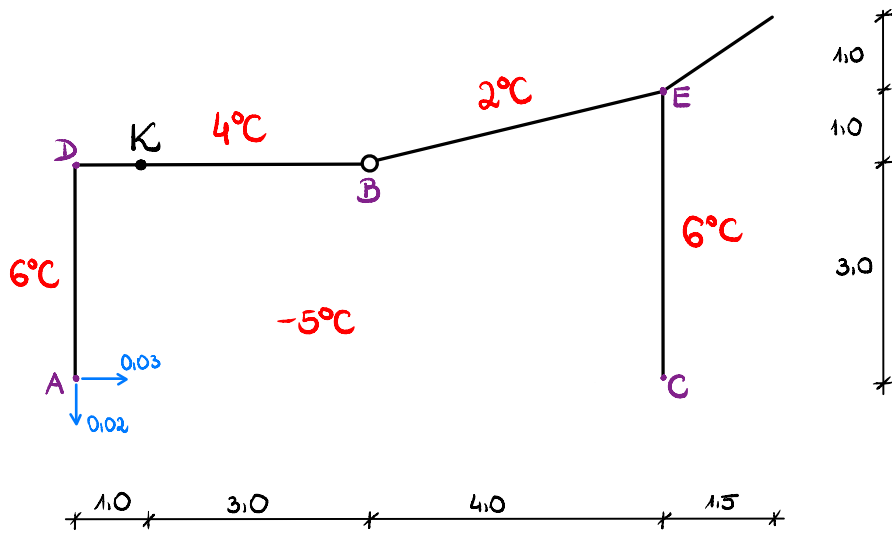


$$0,4851 \sin \alpha + 0,3942 \cos \alpha = 0,5 \text{ kN}$$

$$0,4851 \cos \alpha - 0,3942 \sin \alpha = 0,375$$



STRONA 8



$$\alpha t = 112 \cdot 10^{-5} \frac{1}{^\circ\text{C}}$$

$$t_m = 12^\circ\text{C}$$

$$t_c = \frac{t_2 + t_1}{2}$$

$$t_0 = t_c - t_m$$

$$\Delta t = [t_1 - t_2]$$

$$AD \quad \Delta t = |-5 - 6| = 11^\circ$$

$$t_c = \frac{6 - 5}{2} = 0,5^\circ\text{C}$$

$$t_0 = 0,5^\circ\text{C} - 12^\circ\text{C} = -11,5^\circ\text{C}$$

$$BE \quad \Delta t = |-5 - 2| = 7^\circ$$

$$t_c = \frac{2 - 5}{2} = -1,5^\circ\text{C}$$

$$t_0 = -1,5^\circ\text{C} - 12^\circ\text{C} = -13,5^\circ\text{C}$$

$$DB \quad \Delta t = |-5 - 4| = 9^\circ$$

$$t_c = \frac{4 - 5}{2} = -0,5^\circ\text{C}$$

$$t_0 = -0,5^\circ\text{C} - 12^\circ\text{C} = -12,5^\circ\text{C}$$

$$CE \quad \Delta t = |-5 - 6| = 11^\circ$$

$$t_c = \frac{6 - 5}{2} = 0,5^\circ\text{C}$$

$$t_0 = 0,5^\circ\text{C} - 12^\circ\text{C} = -11,5^\circ\text{C}$$



### Profil dwuteowy (IN; IPE; HEB; HEA)

$$M_y = 53,4 \text{ kN}\cdot\text{m} = 5340 \text{ kN}\cdot\text{cm}$$

$$\sigma \leq 200 \text{ MPa}$$

$$\sigma = \frac{M_y}{W_y} \quad W_y = \frac{M_y}{\sigma} = \frac{5340}{200} = 267 \text{ cm}^3$$

$$\nu = 0,3$$

$$E = 205 \text{ GPa}$$

$$G = \frac{E}{2(1+\nu)} = \frac{205}{2(1+0,3)} = 78,846 \text{ GPa}$$

### Dwuteownik normalny IN 220

$$h = 22 \text{ cm} \quad b = 9,8 \text{ cm} \quad J_y = 3060 \text{ cm}^4 \quad W_y = 278 \text{ cm}^3 \quad A = 39,50 \text{ cm}^2$$

$$EJ = 205 \text{ GPa} \cdot 3060 \text{ cm}^4 = 205 \cdot 10^3 \text{ MPa} \cdot 3060 \text{ cm}^4 = 20,5 \cdot 10^3 \frac{\text{kN}}{\text{cm}^2} \cdot 3060 \text{ cm}^4 = 62730 \cdot 10^3 \text{ kN}\cdot\text{cm}^2 = 6273 \text{ kN}\cdot\text{m}^2$$

$$GA = 78,846 \text{ GPa} \cdot 39,50 \text{ cm}^2 = 78846 \text{ MPa} \cdot 39,50 \text{ cm}^2 = 7884,6 \frac{\text{kN}}{\text{cm}^2} \cdot 39,50 \text{ cm}^2 = 311442 \text{ kN}$$

$$EA = 205 \text{ GPa} \cdot 39,50 \text{ cm}^2 = 205 \cdot 10^3 \text{ MPa} \cdot 39,50 \text{ cm}^2 = 20,5 \frac{\text{kN}}{\text{cm}^2} \cdot 10^3 \cdot 39,50 \text{ cm}^2 = 809750 \text{ kN}$$

$$k = \frac{A}{A_{sr}} = \frac{39,50}{(22 - 2 \cdot 1,22) \cdot 0,81} = 2,493$$

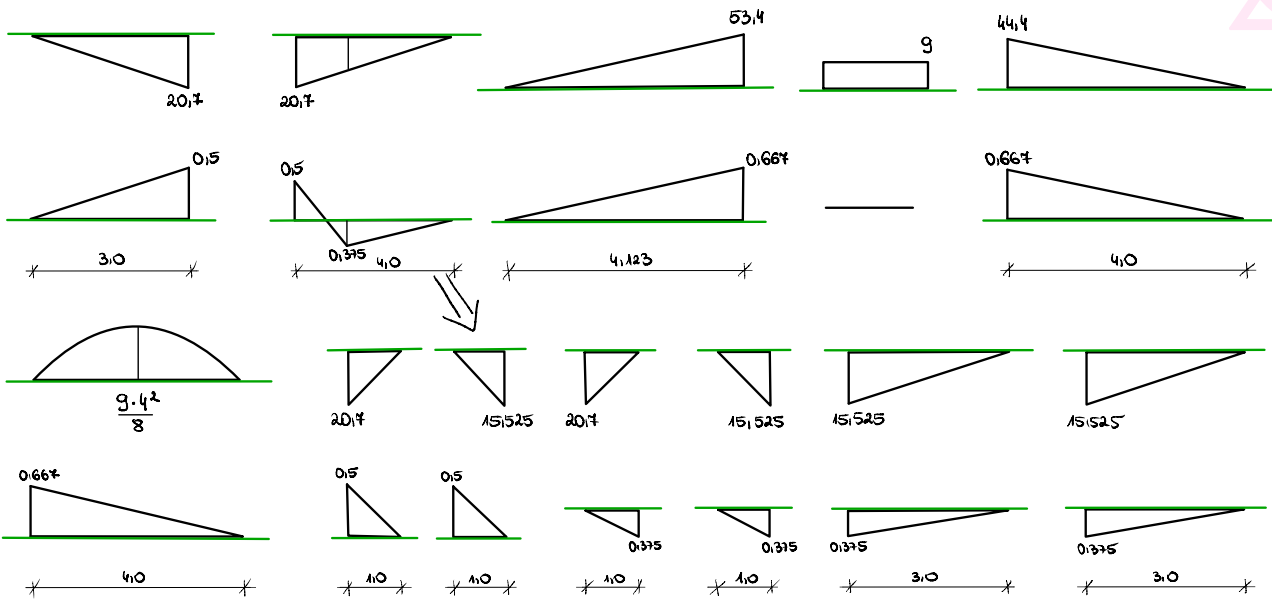
$$t_w = 0,81 \text{ cm}$$

$$t_f = 1,22 \text{ cm}$$



# PIONOWE

$$\varepsilon \int \frac{\bar{M}}{EI} dx$$



$$\begin{aligned} \varepsilon \int \frac{\bar{M}}{EI} dx &= \frac{1}{EI} \left[ -\frac{1}{2} \cdot 3 \cdot 20.7 \cdot \frac{2}{3} \cdot 0.5 + \frac{1}{2} \cdot 4.123 \cdot 53.4 \cdot \frac{2}{3} \cdot 0.667 + \frac{1}{2} \cdot 4.0 \cdot 44.4 \cdot \frac{2}{3} \cdot 0.667 + \frac{2}{3} \cdot \frac{9.4^2}{8} \cdot 4 \cdot \frac{1}{2} \cdot 0.667 - \right. \\ &\quad - \frac{1}{2} \cdot 1.0 \cdot 20.7 \cdot \frac{2}{3} \cdot 0.5 - \frac{1}{2} \cdot 1.0 \cdot 15.525 \cdot \frac{2}{3} \cdot 0.5 + \frac{1}{2} \cdot 1.0 \cdot 20.7 \cdot \frac{2}{3} \cdot 0.375 + \frac{1}{2} \cdot 1.0 \cdot 15.525 \cdot \frac{2}{3} \cdot 0.375 + \frac{1}{2} \cdot 3.0 \cdot 15.525 \cdot \frac{2}{3} \cdot 0.375 + \\ &\quad \left. + \frac{1}{2} \cdot 3.0 \cdot 15.525 \cdot \frac{2}{3} \cdot 0.375 \right] = \frac{104.23}{EI} = \frac{104.23}{6243} = 0.01662 \text{ m} \end{aligned}$$

STRONA 11

# PIONOWE



$$\bar{\Gamma}_B = \varepsilon \int \frac{\bar{M}}{EI} dx + \varepsilon \int \frac{\bar{M} \Delta t}{h} dx + \varepsilon \int \bar{N} \alpha t_0 dx - \varepsilon \bar{R} \Delta$$

$$\varepsilon \int \frac{\bar{M}}{EI} dx = 0.01662 \text{ m}$$

$$\begin{aligned} \varepsilon \int \frac{\bar{M} \Delta t}{h} dx &= \frac{1.2 \cdot 10^{-5}}{0.22} \left[ \frac{1}{2} \cdot 0.5 \cdot 3 \cdot 11 + \frac{1}{2} \cdot 1 \cdot 0.5 \cdot 9 - \frac{1}{2} \cdot 1 \cdot 0.375 \cdot 9 - \frac{1}{2} \cdot 3 \cdot 0.375 \cdot 9 + \right. \\ &\quad \left. + \frac{1}{2} \cdot 4.123 \cdot 0.667 \cdot 7 + \frac{1}{2} \cdot 4 \cdot 0.667 \cdot 11 \right] = \frac{1.2 \cdot 10^{-5}}{0.22} \cdot 28.049 = 0.00153 \text{ m} \end{aligned}$$

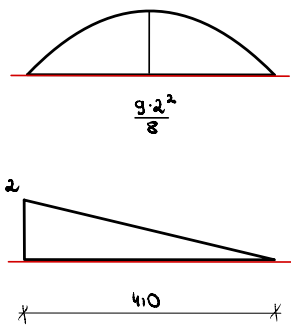
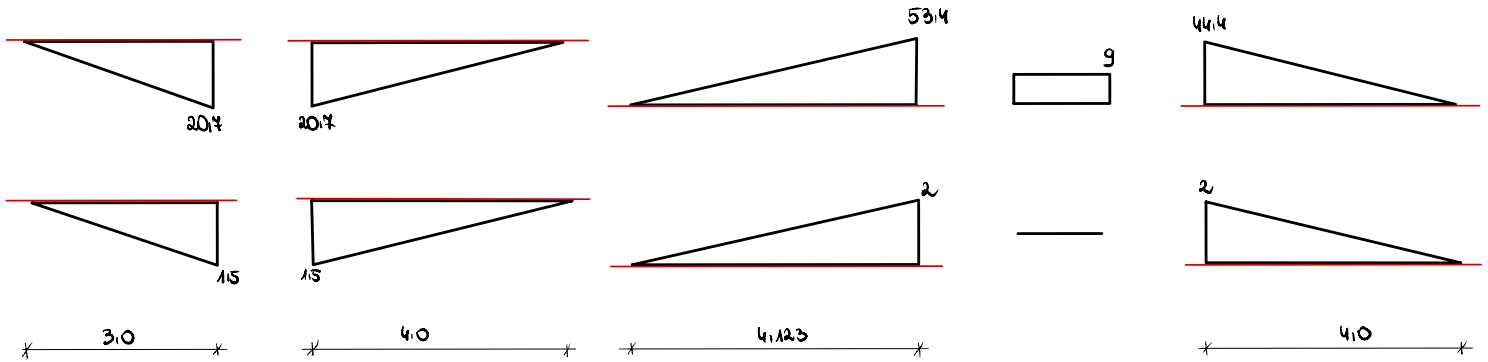
$$\begin{aligned} \varepsilon \int \bar{N} \alpha t_0 dx &= 1.2 \cdot 10^{-5} \left[ -0.875 \cdot 3 \cdot (-11.5) - 0.1667 \cdot 4 \cdot (-12.5) - 0.1314 \cdot 4.123 \cdot \right. \\ &\quad \left. \cdot (-13.5) - 0.125 \cdot 4 \cdot (-11.5) \right] = 1.2 \cdot 10^{-5} \cdot 51.586 = 0.000619 \text{ m} \end{aligned}$$

$$\varepsilon \bar{R} \Delta = -0.875 \cdot 0.02 + 0.1667 \cdot 0.03 = -0.0125 \text{ m}$$

$$\bar{\gamma}^k = 0.01662 + 0.00153 + 0.000619 + 0.0125 = 0.031269 \text{ m} = 3.127 \text{ cm}$$

STRONA 12

# POZIOME



$$\sum \int \frac{MM}{EI} dx = \frac{1}{EI} \left[ +\frac{1}{2} \cdot 3 \cdot 20,7 \cdot \frac{2}{3} \cdot 15 + \frac{1}{2} \cdot 4,0 \cdot 20,7 \cdot \frac{2}{3} \cdot 15 + \right. \\ \left. + \frac{1}{2} \cdot 4,123 \cdot 53,4 \cdot \frac{2}{3} \cdot 2 + \frac{1}{2} \cdot 4,0 \cdot 44,4 \cdot \frac{2}{3} \cdot 2 + \right. \\ \left. + \frac{2}{3} \cdot \frac{9 \cdot 4^2}{8} \cdot 4 \cdot \frac{1}{2} \cdot 2 \right] = \frac{1}{EI} [385,629] = \\ = \frac{385,629}{6273} = 0,06147 \text{ m}$$



STRONA 13

# POZIOME

$$\bar{1} b = \sum \int \frac{MM}{EI} dx + \sum \frac{M \Delta t}{h} dx + \sum \bar{N} \alpha_{to} dx - \sum \bar{R}_D$$

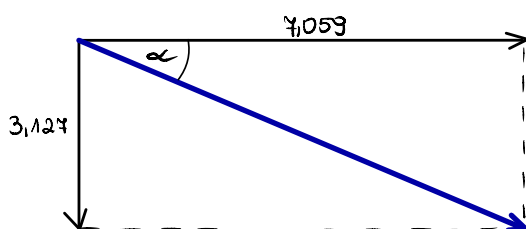
$$\sum \int \frac{MM}{EI} dx = 0,06147 \text{ m}$$

$$\sum \int \frac{M \Delta t}{h} = \frac{1,2 \cdot 10^{-5}}{0,22} \left[ -\frac{1}{2} \cdot 3 \cdot 15 \cdot 11 - \frac{1}{2} \cdot 4 \cdot 15 \cdot 9 + \frac{1}{2} \cdot 4,123 \cdot 2 \cdot 7 + \frac{1}{2} \cdot 4 \cdot 2 \cdot 11 \right] = \frac{1,2 \cdot 10^{-5}}{0,22} [21,11] = 0,001152 \text{ m}$$

$$\sum (\bar{N} \alpha_{to} dx = 1,2 \cdot 10^{-5} [3 \cdot 0,375 \cdot (-11,5) + 0,5 \cdot 1 \cdot (-12,5) - 0,5 \cdot 3 \cdot (-12,5) - 0,3942 \cdot 4,123 \cdot (-13,5) - 0,375 \cdot 4 \cdot (-11,5)] = \\ = 1,2 \cdot 10^{-5} [38,754] = 0,000465$$

$$\sum K = 0,06147 + 0,001152 + 0,000465 + 0,0075 = 0,07059 \text{ m} = 7,059 \text{ cm}$$

Przemieszczenie punktu K (wypadkowa)



$$\sqrt{7,059^2 + 3,127^2} = 7,721 \text{ cm}$$

$$\tan \alpha = \frac{3,127}{7,059} = 0,44298$$

$$\arctg(0,44298) = 23,89^\circ$$



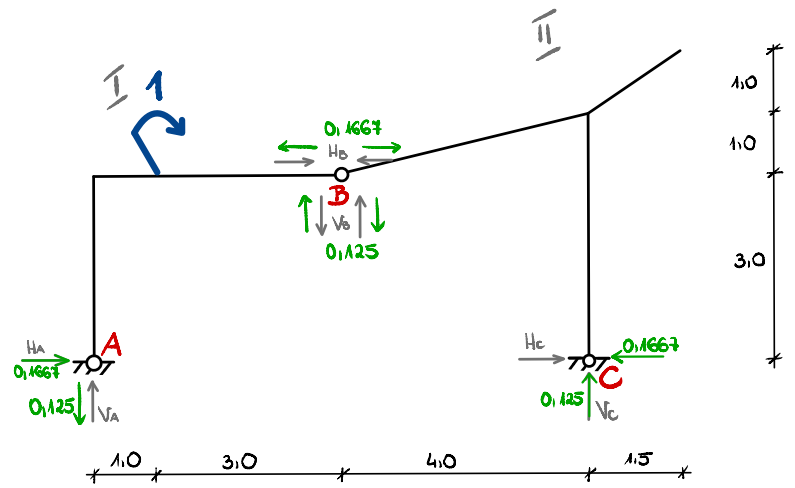
STRONA 14

$$\begin{aligned} \sum M_A = 0 \quad (V_c) \\ -8V_c + 1 = 0 \\ 8V_c = 1 \quad | :8 \\ V_c = 0,125 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum M_C = 0 \quad (V_A) \\ 8V_A + 1 = 0 \\ 8V_A = -1 \quad | :8 \\ V_A = -0,125 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{Spr. } \sum Y = 0 \quad (V_A, V_c) \\ V_A + V_c = 0 \\ 0 = 0 \end{aligned}$$

$$\begin{aligned} \sum M_B \uparrow = 0 \quad (H_A) \\ 4V_A - 3H_A + 1 = 0 \\ 4 \cdot (-0,125) + 1 = 3H_A \quad | :3 \\ H_A = 0,1667 \text{ kN} \end{aligned}$$



$$\begin{aligned} \sum M_B \uparrow = 0 \\ -4V_c - 3H_c = 0 \\ -4 \cdot 0,125 = 3H_c \\ H_c = -0,1667 \text{ kN} \end{aligned}$$

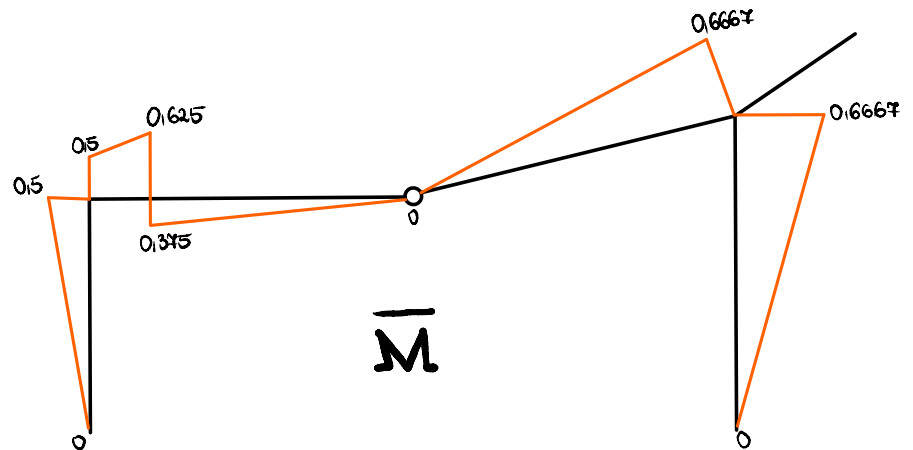
$$\begin{aligned} \sum X \uparrow = 0 \\ H_A + H_B = 0 \\ H_B = -0,1667 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum X = 0 \quad (H_c, H_A) \\ H_A + H_c = 0 \\ 0 = 0 \end{aligned}$$

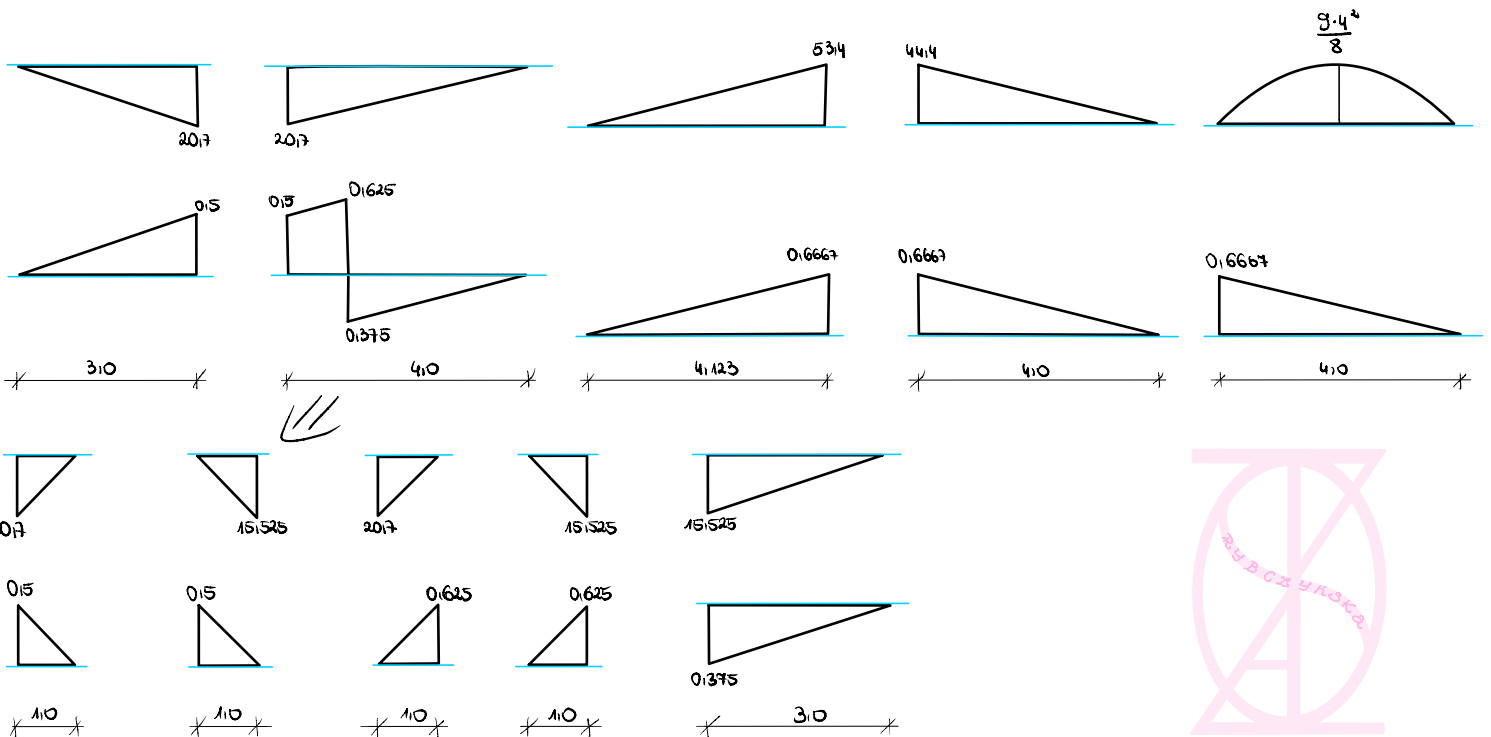
$$\begin{aligned} \sum X \uparrow = 0 \\ -H_B + H_c = 0 \\ H_B = -0,1667 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum Y \downarrow = 0 \\ V_A - V_B = 0 \\ V_B = -0,125 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum Y \uparrow = 0 \\ V_B + V_c = 0 \\ V_B = -V_c \\ V_B = -0,125 \text{ kN} \end{aligned}$$



STRONA 15



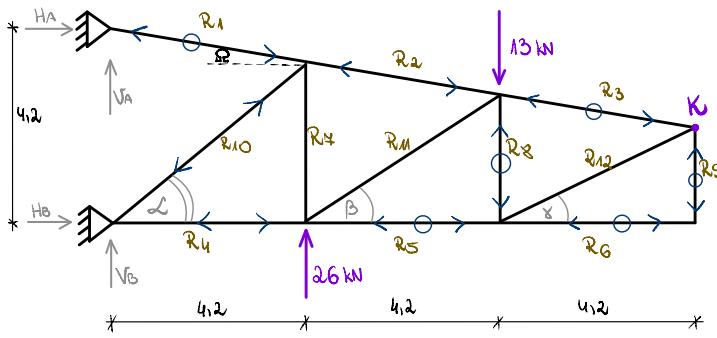
$$\begin{aligned} \varphi = \int \frac{M}{EI} dx = \frac{1}{EI} \left[ -\frac{1}{2} \cdot 3 \cdot 20,7 \cdot \frac{2}{3} \cdot 0,15 + \frac{1}{2} \cdot 4,123 \cdot 53,4 \cdot \frac{2}{3} \cdot 0,6667 + \frac{1}{2} \cdot 4 \cdot 44,4 \cdot \frac{2}{3} \cdot 0,6667 + \frac{2}{3} \cdot \frac{9,4^2}{8} \cdot 4 \cdot \frac{1}{2} \cdot 0,6667 - \frac{1}{2} \cdot 1 \cdot 20,7 \cdot \frac{2}{3} \cdot 0,15 - \right. \\ \left. - \frac{1}{2} \cdot 1 \cdot 15,525 \cdot \frac{2}{3} \cdot 0,15 - \frac{1}{2} \cdot 1 \cdot 20,7 \cdot \frac{2}{3} \cdot 0,625 - \frac{1}{2} \cdot 1 \cdot 15,525 \cdot \frac{2}{3} \cdot 0,625 + \frac{1}{2} \cdot 3 \cdot 15,525 \cdot \frac{2}{3} \cdot 0,375 \right] = \frac{89,736}{6275} = 0,01431 \text{ rad} = 0,8199^\circ \end{aligned}$$

STRONA 16

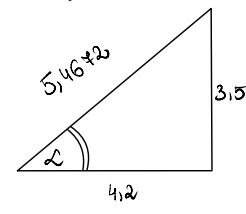


# Kratownica

# Rybczyńska Łośia



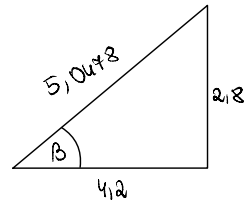
L=1,4  
P=13kN



$$\sqrt{3,5^2 + 4,2^2} = 5,4672$$

$$\sin \alpha = \frac{3,5}{5,4672} = 0,6402$$

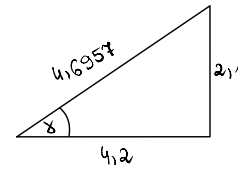
$$\cos \alpha = \frac{4,2}{5,4672} = 0,7682$$



$$\sqrt{2,8^2 + 4,2^2} = 5,0478$$

$$\sin \beta = \frac{2,8}{5,0478} = 0,5547$$

$$\cos \beta = \frac{4,2}{5,0478} = 0,8320$$



$$\sqrt{4,2^2 + 2,1^2} = 4,6957$$

$$\sin \gamma = \frac{2,1}{4,6957} = 0,4472$$

$$\cos \gamma = \frac{4,2}{4,6957} = 0,8944$$

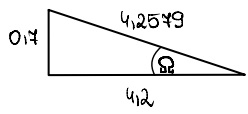


$\sum M_B = 0$   
 $2 \cdot 4,2 \cdot 13 - 4,2 \cdot 26 + 4,2 H_A = 0$   
 $H_A = 0,0 \text{ kN}$

$\sum M_A = 0$   
 $2 \cdot 4,2 \cdot 13 - 4,2 \cdot 26 - 4,2 \cdot H_B = 0$   
 $H_B = 0,0 \text{ kN}$

$\sum M = 0$   
 $4,2 V_B + 4,2 \cdot 13 = 0$   
 $V_B = -13 \text{ kN}$

$\sum Y = 0$   
 $V_B + V_A + 26 - 13 = 0$   
 $V_A = -13 + 13 = 0 \text{ kN}$

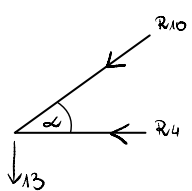


$$\sqrt{4,2^2 + 0,7^2} = 4,2579$$

$$\sin \omega = \frac{0,7}{4,2579} = 0,1644$$

$$\cos \omega = \frac{4,2}{4,2579} = 0,9864$$

STRONA 17

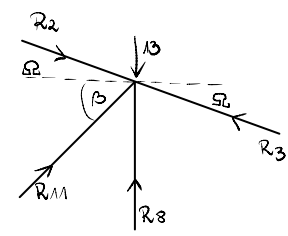


$$R_{10} \sin \alpha + 13 = 0$$

$$R_{10} = \frac{-13}{\sin \alpha} = -20,306 \text{ kN}$$

$$R_{10} \cos \alpha + R_4 = 0$$

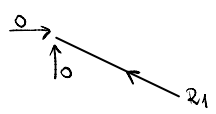
$$R_4 = -R_{10} \cos \alpha = -15,599 \text{ kN}$$



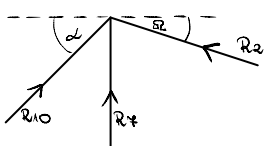
$$R_2 \cos \epsilon + R_{11} \cos \beta - R_3 \cos \delta = 0$$

$$R_3 = \frac{R_2 \cos \epsilon + R_{11} \cos \beta}{\cos \delta} = 0$$

$R_9 = 0 \text{ kN}$   
 $R_6 = 0 \text{ kN}$   
 $R_{12} = 0 \text{ kN}$



$R_1 = 0 \text{ kN}$



$$R_{10} \cos \alpha - R_2 \cos \epsilon = 0$$

$$R_2 = \frac{R_{10} \cos \alpha}{\cos \epsilon} = -15,814 \text{ kN}$$

$$R_7 + R_{10} \sin \alpha + R_2 \sin \epsilon = 0$$

$$R_7 = -R_{10} \sin \alpha - R_2 \sin \epsilon = 15,600 \text{ kN}$$



$$R_8 + R_{11} \sin \beta + R_3 \sin \delta - 13 - R_2 \sin \epsilon = 0$$

$$R_8 = 13 + R_2 \sin \epsilon - R_{11} \sin \beta = 0$$

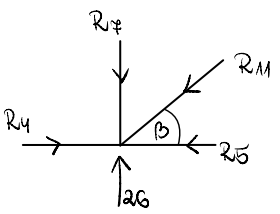
$$-13 \cdot 4,2 + 3,5 \cdot R_4 = 0$$

$$R_4 = \frac{-13 \cdot 4,2}{3,5} = -15,6 \text{ kN}$$

$$-4,2 \cdot 13 - R_2 \cdot 3,5 \cdot \cos \epsilon = 0$$

$$R_2 = \frac{-4,2 \cdot 13}{3,5 \cdot \cos \epsilon} = -15,815 \text{ kN}$$

$$-6 \cdot 4,2 \cdot 13 + 5 \cdot 4,2 \cdot 26 - 18,749 \sin \beta \cdot 5 \cdot 4,2 = 0$$

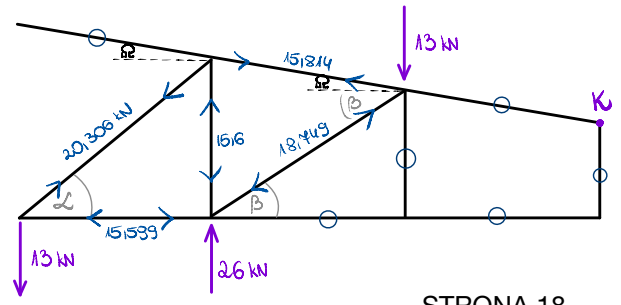


$$26 - R_7 - R_{11} \sin \beta = 0$$

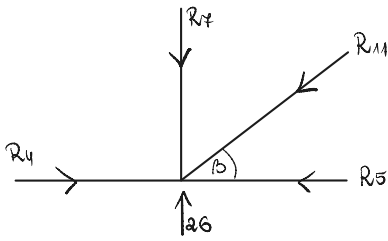
$$R_{11} = \frac{26 - R_7}{\sin \beta} = 18,749 \text{ kN}$$

$$R_4 - R_{11} \cos \beta - R_5 = 0$$

$$R_5 = R_4 - R_{11} \cos \beta = -31,198 \text{ kN}$$



STRONA 18

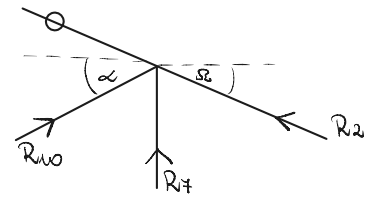


$$R_7 + R_{11} \sin \beta - 26 = 0$$

$$R_{11} = \frac{26 - R_7}{\sin \beta} = 18,748 \text{ kN}$$

$$R_4 - R_{11} \cos \beta - R_5 = 0$$

$$R_5 = R_4 - R_{11} \cos \beta = -0,0002 \text{ kN}$$

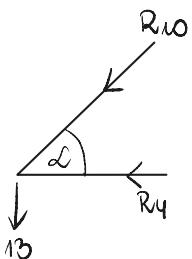


$$R_{10} \cos \alpha - R_2 \cos \beta = 0$$

$$R_2 = \frac{R_{10} \cos \alpha}{\cos \beta} = -15,814 \text{ kN}$$

$$R_7 + R_{10} \sin \alpha + R_2 \sin \beta = 0$$

$$R_7 = -R_{10} \sin \alpha - R_2 \sin \beta = 15,6 \text{ kN}$$

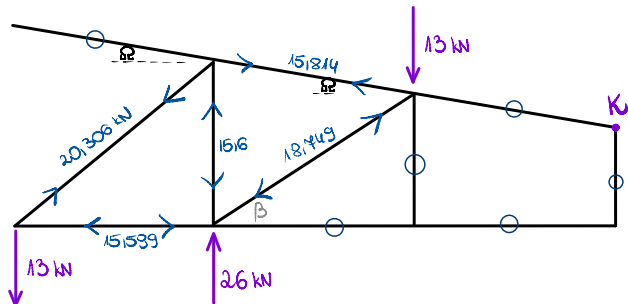


$$13 + R_{10} \sin \alpha = 0$$

$$R_{10} = \frac{-13}{\sin \alpha} = -20,306 \text{ kN}$$

$$R_{10} \cos \alpha + R_4 = 0$$

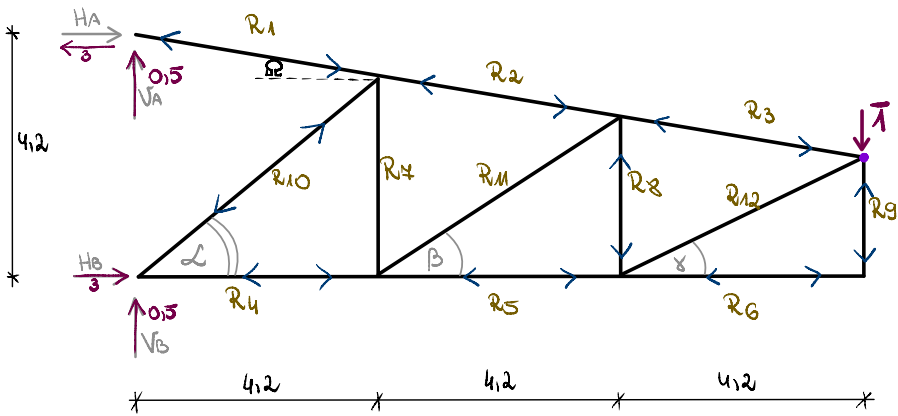
$$R_4 = -R_{10} \cos \alpha = 15,599 \text{ kN}$$



$$R_{10} = -20,306 \text{ kN} \quad R_4 = 15,599 \text{ kN} \quad R_2 = -15,814 \text{ kN}$$

$$R_{11} = 18,748 \text{ kN} \quad R_7 = 15,6 \text{ kN}$$

STRONA 19



$$\sin \alpha = 0,6402$$

$$\cos \alpha = 0,7682$$

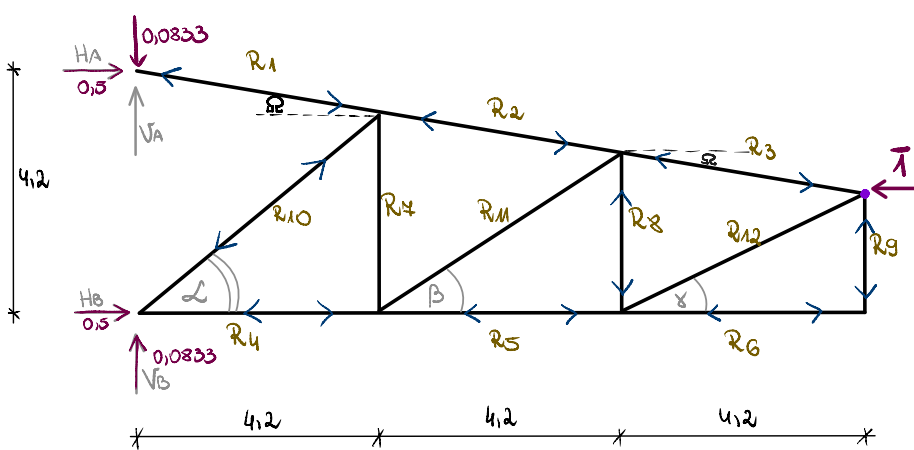


$$\sin \beta = 0,5547$$

$$\cos \beta = 0,8320$$

$$\sin \gamma = 0,4472$$

$$\cos \gamma = 0,8944$$

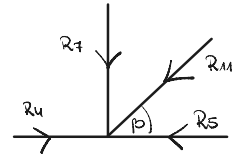


$$\sin \beta = 0,1644$$

$$\cos \beta = 0,9864$$

STRONA 20

# 1 Pionowa



$$R_7 + R_{11} \sin \beta = 0$$

$$R_{11} = -\frac{R_7}{\sin \beta} = 1,082 \text{ kN}$$

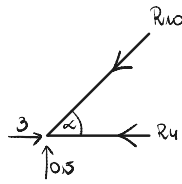
$$R_4 - R_{11} \cos \beta - R_5 = 0$$

$$R_5 = R_4 - R_{11} \cos \beta = 1,5 \text{ kN}$$

$$\sum H_A = 0$$

$$3 \cdot 4,2 \cdot 1 - 4,2 H_B = 0$$

$$H_B = 3 \text{ kN}$$



$$R_{10} = \frac{0,15}{\sin \alpha} = 0,1781$$

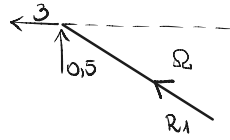
$$3 - R_4 - R_{10} \cos \alpha = 0$$

$$R_4 = 3 - 0,1781 \cdot 0,7682 = 2,4 \text{ kN}$$

$$\sum M = 0$$

$$4,2 H_B - 3,5 H_B + 2 \cdot 4,2 \cdot 1 = 0$$

$$H_B = \frac{3,5 H_B - 2 \cdot 4,2}{4,2} = 0,15 \text{ kN}$$

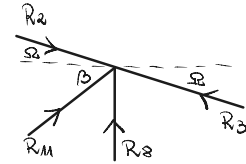


$$3 + R_1 \cos \alpha = 0$$

$$R_1 = -\frac{3}{\cos \alpha} = -3,041 \text{ kN}$$

$$0,15 + R_1 \sin \alpha = 0$$

$$R_1 = -\frac{0,15}{\sin \alpha} = -3,041 \text{ kN}$$



$$R_2 \cos \beta + R_{11} \cos \beta - R_3 \cos \beta = 0$$

$$R_3 = \frac{R_2 \cos \beta + R_{11} \cos \beta}{\cos \beta} = -1,152 \text{ kN}$$

$$R_8 + R_{11} \sin \beta + R_3 \sin \beta - R_2 \sin \beta = 0$$

$$R_8 = R_2 \sin \beta - R_{11} \sin \beta - R_3 \sin \beta = -0,175 \text{ kN}$$

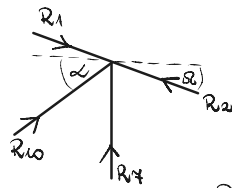
$$\sum Y = 0$$

$$V_A = 0,5 \text{ kN}$$

$$\sum M = 0$$

$$4,2 V_A + 0,17 H_A = 0$$

$$V_A = -\frac{0,17}{4,2} \cdot H_A = 0,5 \text{ kN}$$



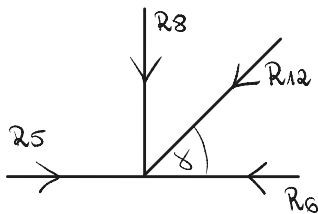
$$R_1 \cos \alpha + R_{10} \cos \alpha - R_2 \cos \alpha = 0$$

$$R_2 = \frac{R_1 \cos \alpha + R_{10} \cos \alpha}{\cos \alpha} = -2,433 \text{ kN}$$

$$R_{10} \sin \alpha + R_7 + R_2 \sin \beta - R_1 \sin \alpha = 0$$

$$R_7 = R_1 \sin \alpha - R_{10} \sin \alpha - R_2 \sin \beta = -0,6 \text{ kN}$$

## STRONA 21



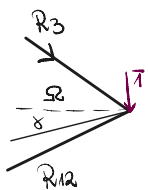
$$R_5 - R_6 - R_{12} \cos \gamma = 0$$

$$R_8 + R_{12} \sin \gamma = 0$$

$$R_{12} = -\frac{R_8}{\sin \gamma} = \frac{0,175}{0,14472} = 1,677 \text{ kN}$$

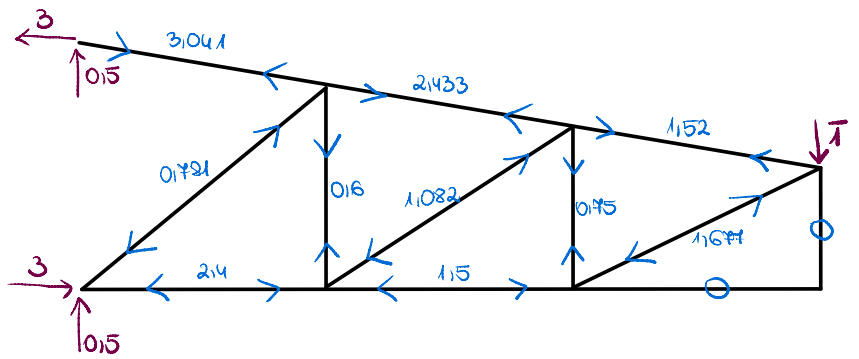
$$R_9 = R_5 - R_{12} \cos \gamma = 0$$

$$R_9 = 0 \text{ kN}$$



$$R_{12} \cos \gamma + R_3 \cos \beta = 0$$

$$R_{12} \sin \gamma - R_3 \sin \beta - 1 = 0$$



$$R_1 = -3,041$$

$$R_2 = -2,433$$

$$R_3 = -1,152$$

$$R_4 = 2,4$$

$$R_5 = 1,15$$

$$R_6 = 0$$

$$R_7 = -0,6$$

$$R_8 = -0,175$$

$$R_9 = 0$$

$$R_{10} = 0,1781$$

$$R_{11} = 1,082$$

$$R_{12} = 1,677$$

$$L_1 = 4,2579 \text{ m}$$

$$L_2 = 4,2579 \text{ m}$$

$$L_3 = 4,2579 \text{ m}$$

$$L_4 = 4,2 \text{ m}$$

$$L_5 = 4,2 \text{ m}$$

$$L_6 = 4,2 \text{ m}$$

$$L_7 = 3,5 \text{ m}$$

$$L_8 = 2,8 \text{ m}$$

$$L_9 = 2,1 \text{ m}$$

$$L_{10} = 5,4672 \text{ m}$$

$$L_{11} = 5,0478 \text{ m}$$

$$L_{12} = 4,6957 \text{ m}$$

## STRONA 22

$$\sum M_B = 0$$

$$4,2 H_A - 2,1 \cdot 1 = 0$$

$$H_A = \frac{2,1}{4,2} = 0,5 \text{ kN}$$

$$\sum M_A = 0$$

$$1 \cdot 2,1 - 4,2 H_B = 0$$

$$H_B = \frac{2,1}{4,2} = 0,5 \text{ kN}$$

$$\sum M = 0$$

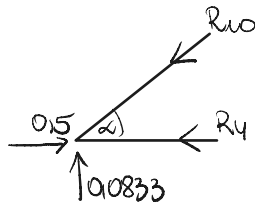
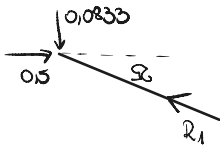
$$1,4 \cdot 1 + 4,2 V_B - 3,5 H_B = 0$$

$$V_B = \frac{(3,5 \cdot 0,5 - 1,4)}{4,2} = 0,0833 \text{ kN}$$

$$\sum Y = 0$$

$$4,2 V_A + 0,1 H_A = 0$$

$$V_A = \frac{-0,1 \cdot 0,5}{4,2} = -0,0833 \text{ kN}$$



$$0,0833 - R_{10} \sin \alpha = 0$$

$$R_{10} = \frac{0,0833}{\sin \alpha} = 0,13 \text{ kN}$$

$$0,15 - R_4 - R_{10} \cos \alpha = 0$$

$$R_4 + 0,15 - 0,13 \cos \alpha = 0,4$$

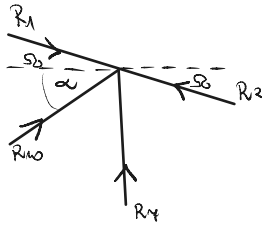
$$0,15 - R_1 \cos \alpha = 0$$

$$R_1 = \frac{0,15}{\cos \alpha} = 0,5069$$

$$R_1 \sin \alpha - 0,0833 = 0$$

$$R_1 = \frac{0,0833}{\sin \alpha} = 0,5069 \text{ kN}$$

## Przykład

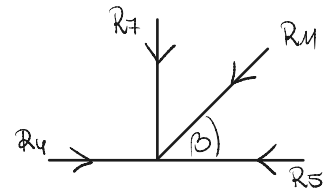


$$R_1 \cos \alpha + R_{10} \cos \alpha - R_2 \cos \alpha = 0$$

$$R_2 = \frac{R_1 \cos \alpha + R_{10} \cos \alpha}{\cos \alpha} = 0,6081 \text{ kN}$$

$$R_{10} \sin \alpha + R_7 + R_2 \sin \alpha - R_1 \sin \alpha = 0$$

$$R_7 = R_1 \sin \alpha - R_{10} \sin \alpha - R_2 \sin \alpha = -0,0999 \text{ kN}$$

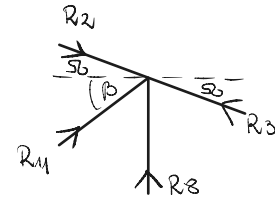


$$R_7 + R_{11} \sin \beta = 0$$

$$R_{11} = \frac{-R_7}{\sin \beta} = 0,18 \text{ kN}$$

$$R_4 - R_5 - R_{11} \cos \beta = 0$$

$$R_5 = R_4 - R_{11} \cos \beta = 0,4 - 0,18 \cos \beta = 0,2502 \text{ kN}$$



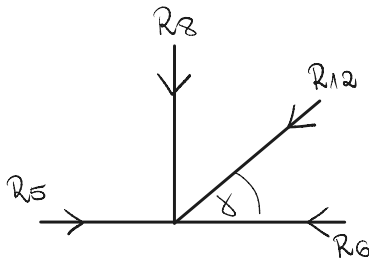
$$R_2 \cos \alpha + R_{11} \cos \beta + R_3 \cos \alpha = 0$$

$$R_3 = \frac{-R_2 \cos \alpha - R_{11} \cos \beta}{\cos \alpha} = 0,76 \text{ kN}$$

$$R_8 + R_{11} \sin \beta + R_3 \sin \alpha - R_2 \sin \alpha = 0$$

$$R_8 = R_2 \sin \alpha - R_{11} \sin \beta - R_3 \sin \alpha = -0,1402 \text{ kN}$$

STRONA 23

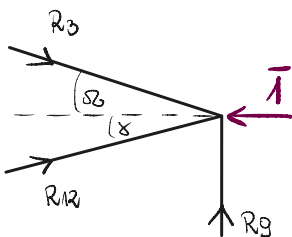


$$R_8 + R_{12} \sin \gamma = 0$$

$$R_{12} = \frac{-R_8}{\sin \gamma} = 0,3135 \text{ kN}$$

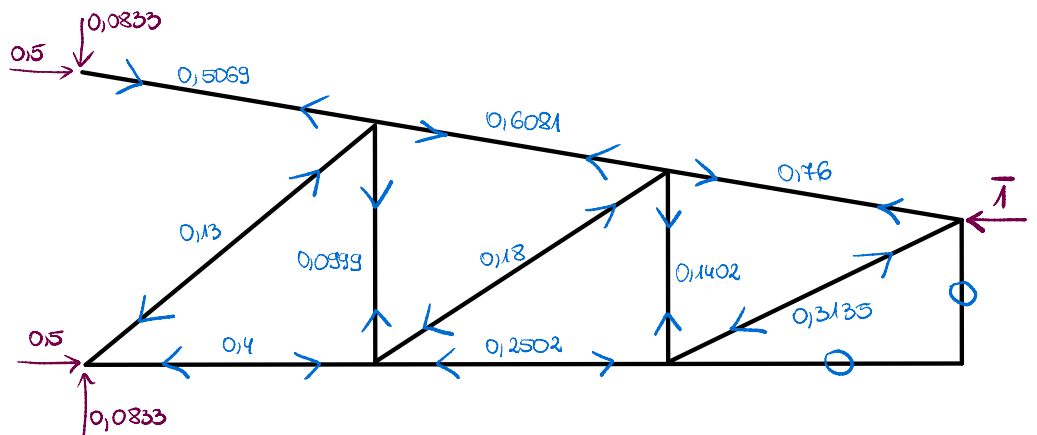
$$R_5 - R_{12} \cos \gamma - R_6 = 0$$

$$-0,03 = 0$$



$$R_3 \cos \alpha + R_{12} \cos \gamma - 1 = 0$$

$$0,03 = 0$$



$$R_1 = 0,5069$$

$$R_2 = 0,6081$$

$$R_3 = 0,76$$

$$R_4 = 0,4$$

$$R_5 = 0,2502$$

$$R_6 = 0$$

$$R_7 = -0,0999$$

$$R_8 = -0,1402$$

$$R_9 = 0$$

$$R_{10} = 0,13$$

$$R_{11} = 0,18$$

$$R_{12} = 0,3135$$



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# Kształownik zamknięty kwadratowy

$$\sigma_x \leq 200 \text{ MPa}$$

$$N_{\max} = 20,306 \text{ kN}$$

$$\sigma_x = \frac{20 \text{ kN}}{\text{cm}^2} = \frac{N}{A}$$

$$A = \frac{N}{20} = 1,015 \text{ cm}^2$$



# Przyjęty kształownik

$$40 \times 40 \text{ mm}$$

$$t = 2,5 \text{ mm}$$

$$A = 2,89 \text{ cm}^2$$

$$E = 205 \text{ GPa}$$

$$EA = 205 \cdot 10^3 \text{ MPa} \cdot 2,89 \text{ cm}^2 =$$

$$= 20,5 \cdot 10^3 \frac{\text{kN}}{\text{cm}^2} \cdot 2,89 =$$

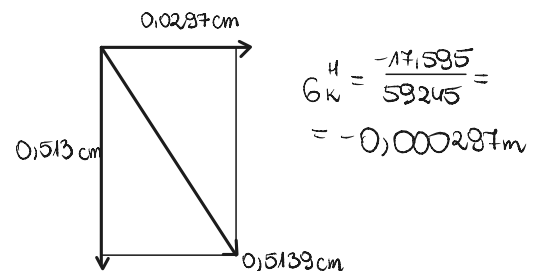
$$= 59,245 \cdot 10^3 \text{ kN} = 59245 \text{ kN}$$

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Lp.	Długość	N [kN]	Pion $\bar{N} [F]$	$\bar{N} \bar{L}$	$\bar{N} [F]$ Poziom	$\bar{N} \bar{L}$ Poziom
1	4,2579	0	-3,041	0	0,3069	0
2	4,2579	-15,814	-2,1433	163,82	0,6081	-40,946
3	4,2579	0	-1,52	0	0,76	0
4	4,2	15,599	2,4	157,24	0,4	26,206
5	4,2	0	1,5	0	0,2502	0
6	4,2	0	0	0	0	0
7	3,5	15,6	-0,6	-32,76	-0,1	-5,46
8	2,8	0	-0,75	0	-0,1402	0
9	2,1	0	0	0	0	0
10	5,4672	-20,306	0,721	-86,70	0,13	-14,43
11	5,0478	18,749	1,082	102,140	0,18	17,035
12	4,6957	0	1,677	0	0,315	0
			$\Sigma \bar{N} \bar{L}$	304		-17,595

$$\delta_K^V = \frac{304}{59245} = 0,00513 \text{ m}$$

$$\sqrt{0,513^2 + 0,0287^2} = 0,5139 \text{ m}$$



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