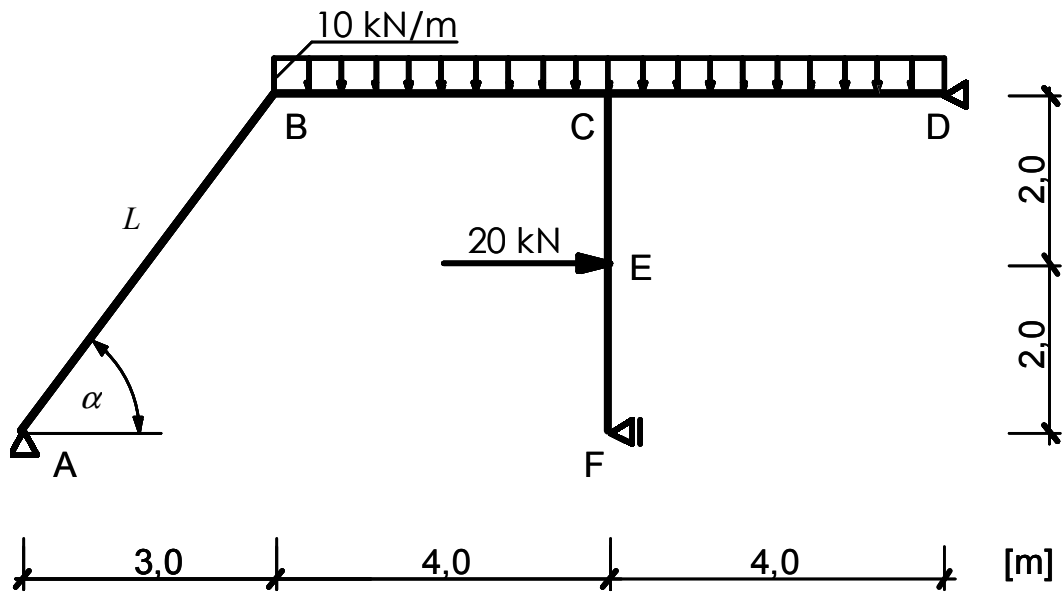


POLITECHNIKA POZNAŃSKA

Metoda sił.

Stefan Wyczkowski
grupa BZ-21
rok II – semestr I
rok akademicki 2006/07

SCHEMAT STATYCZNY ZADANEJ RAMY WRAZ Z OBCIĄŻENIEM

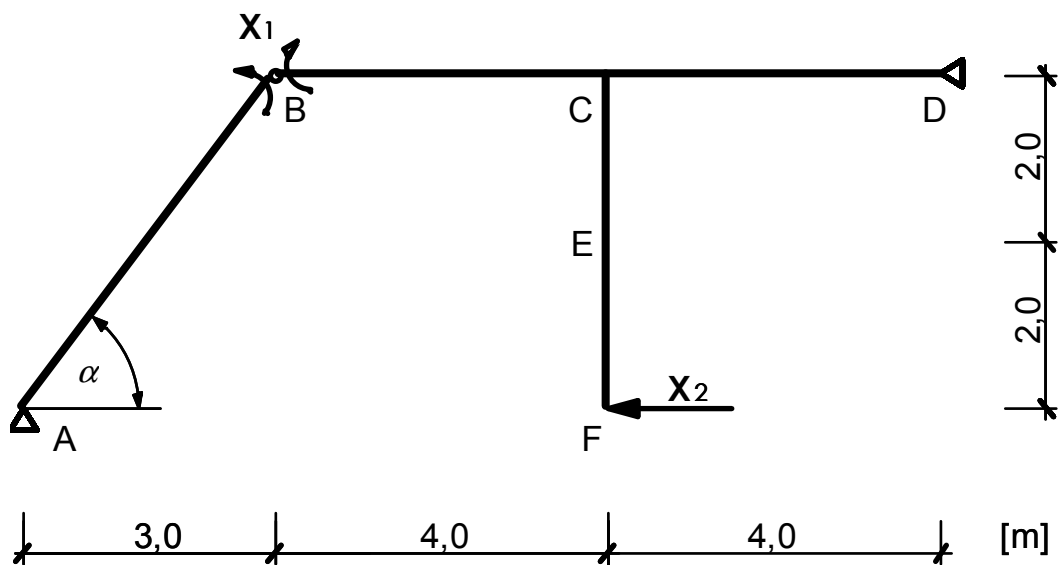


SSN = 2

$$\operatorname{tg} \alpha = \frac{4,0}{3,0} \Rightarrow \alpha = 53,13^\circ$$

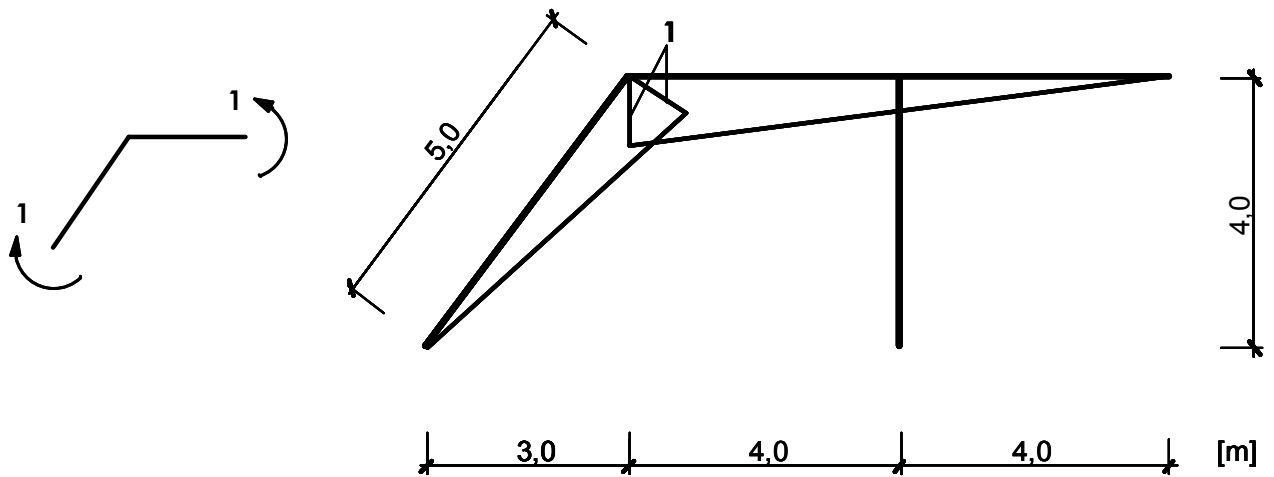
$$L = \sqrt{3,0^2 + 4,0^2} \Rightarrow L = 5,0 \text{ m}$$

PRZYJĘTO NASTĘPUJĄCY UKŁAD PODSTAWOWY

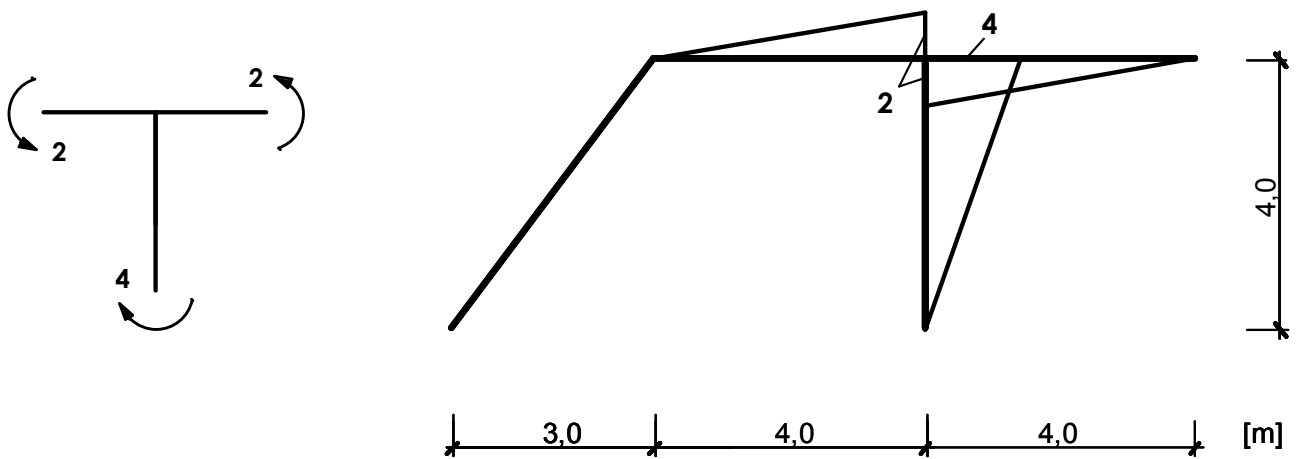


WYKRESY MOMENTÓW DLA STANÓW „1”, „2”, „P”

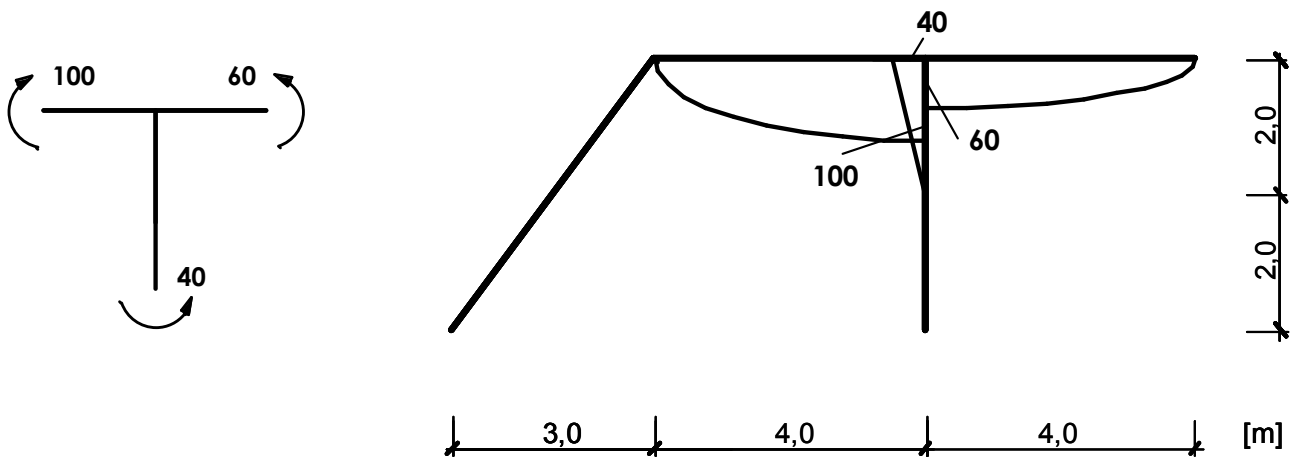
„1” → $X_1 = 1$ | $X_2 = 0$ | $obc = 0$



„2” → $X_1 = 0$ | $X_2 = 1$ | $obc = 0$



„P” → $X_1 = 0$ | $X_2 = 0$ | $obc \neq 0$



**OBLICZANIE SKŁADOWYCH MACIERZY PODATNOŚCI
ORAZ MACIERZY WYRAZÓW WOLNYCH**

$$\delta_{11} = \frac{1}{EI} \left[\frac{1}{2} \cdot 5 \cdot 1 \cdot \frac{2}{3} \cdot 1 + \frac{1}{2} \cdot 8 \cdot 1 \cdot \frac{2}{3} \cdot 1 \right] = \frac{4,333}{EI}$$

$$\delta_{22} = \frac{1}{EI} \left[\left(\frac{1}{2} \cdot 4 \cdot 2 \cdot \frac{2}{3} \cdot 2 \right) \cdot 2 + \frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 4 \right] = \frac{32,0}{EI}$$

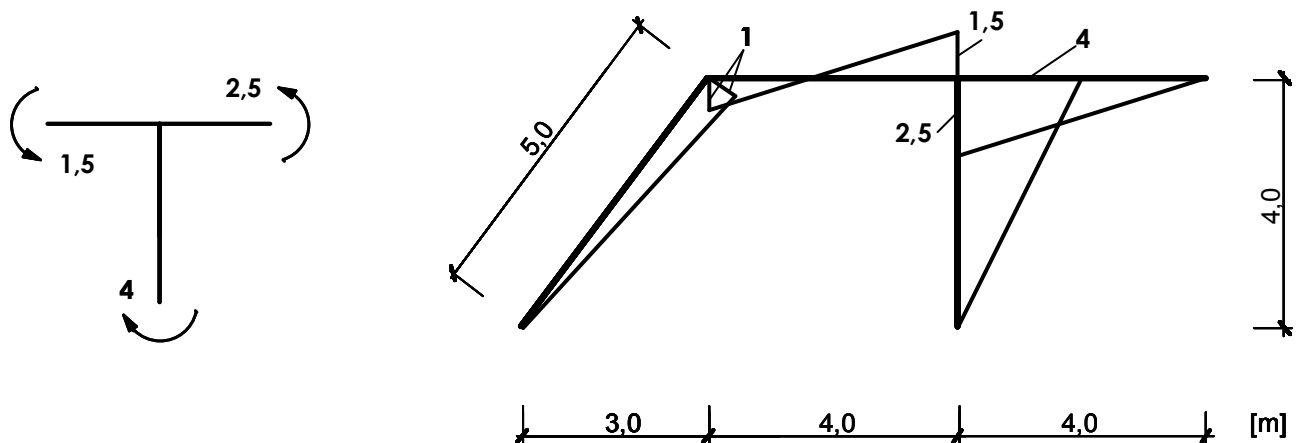
$$\delta_{12} = \delta_{21} = \frac{1}{EI} \left[\frac{4}{6} \cdot \left(-2 \cdot \frac{1}{2} \cdot 2 - 1 \cdot 2 \right) + \frac{1}{2} \cdot 4 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot 2 \right] = -\frac{1,333}{EI}$$

$$\delta_{1P} = \frac{1}{EI} \left[\frac{2}{3} \cdot 4 \cdot 20 \cdot \frac{1}{2} \cdot \left(1 + \frac{1}{2} \right) + \frac{4}{6} \cdot \left(2 \cdot \frac{1}{2} \cdot 100 + 1 \cdot 100 \right) + \frac{2}{3} \cdot 4 \cdot 20 \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot 4 \cdot \frac{2}{3} \cdot 60 \right] = \frac{226,667}{EI}$$

$$\delta_{2P} = \frac{1}{EI} \left[-\frac{1}{2} \cdot 4 \cdot 2 \cdot \frac{2}{3} \cdot 100 + \frac{1}{2} \cdot 4 \cdot 2 \cdot \frac{2}{3} \cdot 60 + \frac{2}{6} \cdot \left(-2 \cdot 4 \cdot 40 - 2 \cdot 40 \right) \right] = -\frac{240,0}{EI}$$

SPRAWDZENIE POPRAWNOŚCI OTRZYMANYCH WARTOŚCI

"S" → $X_1 = 1$ | $X_2 = 1$ | $obc = 0$



$$\delta_{SS} = \frac{1}{EI} \left[\frac{1}{2} \cdot 5 \cdot 1 \cdot \frac{2}{3} \cdot 1 + \frac{4}{6} \cdot (2 \cdot 1 \cdot 1 + 2 \cdot 1,5 \cdot 1,5 - 2 \cdot 1 \cdot 1,5) + \frac{1}{2} \cdot 4 \cdot 2,5 \cdot \frac{2}{3} \cdot 2,5 + \frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 4 \right] = \frac{33,667}{EI}$$

$$\delta_{SP} = \frac{1}{EI} \left[\frac{2}{3} \cdot 4 \cdot 20 \cdot \frac{1}{2} \cdot (1 - 1,5) + \frac{4}{6} \cdot (-2 \cdot 1,5 \cdot 100 + 1 \cdot 100) + \frac{2}{3} \cdot 4 \cdot 20 \cdot \frac{1}{2} \cdot 2,5 + \frac{1}{2} \cdot 4 \cdot 60 \cdot \frac{2}{3} \cdot 2,5 + \frac{2}{6} \cdot (-2 \cdot 4 \cdot 40 - 2 \cdot 40) \right] = -\frac{13,333}{EI}$$

$$\delta_{SS} = \delta_{11} + \delta_{22} + \delta_{12} + \delta_{21}$$

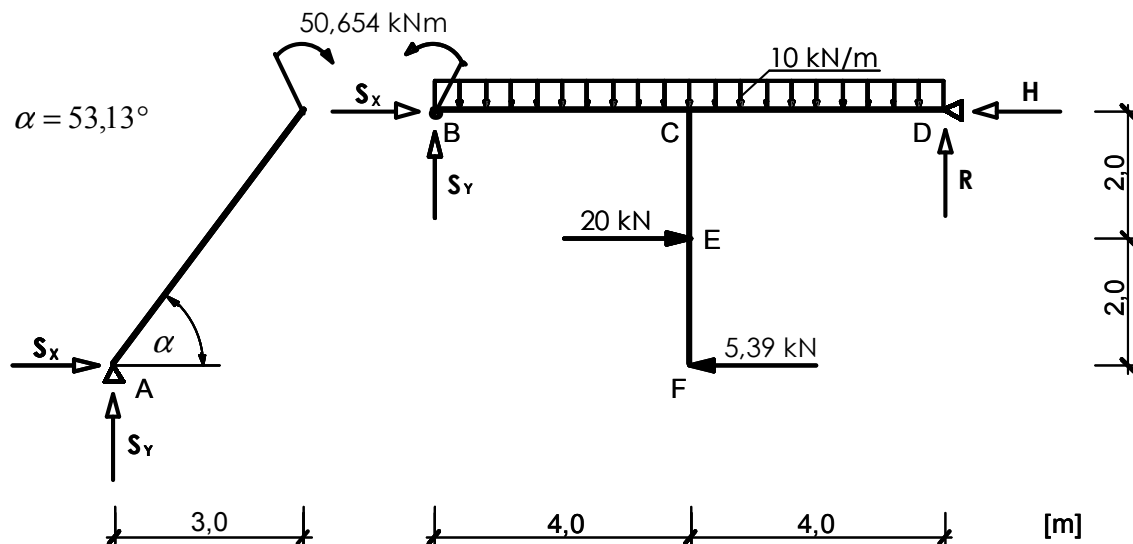
$$\delta_{SP} = \delta_{1P} + \delta_{2P}$$

OBLICZANIE NIEWIADOMYCH X_1 ORAZ X_2
PRZY WYKORZYSTANIU RÓWNIANIA MACIERZOWEGO

$$\begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} \cdot \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} + \begin{Bmatrix} \delta_{1P} \\ \delta_{2P} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \rightarrow \quad \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = - \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}^{-1} \cdot \begin{Bmatrix} \delta_{1P} \\ \delta_{2P} \end{Bmatrix}$$

$$\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = - \begin{bmatrix} \frac{0,234}{EI} & \frac{0,01}{EI} \\ \frac{0,01}{EI} & \frac{0,032}{EI} \end{bmatrix} \cdot \begin{Bmatrix} \frac{226,667}{EI} \\ -\frac{240,0}{EI} \end{Bmatrix} \quad \rightarrow \quad \begin{array}{l} X_1 = -50,654 \text{ kN} \\ X_2 = 5,390 \text{ kN} \end{array}$$

OBLICZANIE NIEWIADOMYCH REAKCJI S , R , H



$$\sum M_B^P = -R_D \cdot 8,0 + 10,0 \cdot 8,0 \cdot 4,0 - 20,0 \cdot 2,0 + 5,39 \cdot 4,0 - 50,654 = 0$$

$$\rightarrow R_D = 31,363 \text{ kN}$$

$$\sum M_D = S_Y \cdot 8,0 - 50,654 - 10,0 \cdot 8,0 \cdot 4,0 - 20,0 \cdot 2,0 + 5,39 \cdot 4,0 = 0$$

$$\rightarrow S_Y = 48,637 \text{ kN}$$

$$\sum M_B^L = -S_X \cdot 4,0 + 48,637 \cdot 3,0 + 50,654 = 0$$

$$\rightarrow S_X = 49,141 \text{ kN}$$

$$\sum X = -H_D + 49,141 + 20,0 - 5,39 = 0$$

$$\rightarrow H_D = 63,751 \text{ kN}$$

WYZNACZANIE SIŁ WEWNĘTRZNYCH W RAMIE

ODCINEK AB

$$N = -49,141 \cdot \cos \alpha - 48,637 \cdot \sin \alpha = -68,394 \text{ kN}$$

$$T = -49,141 \cdot \sin \alpha + 48,637 \cdot \cos \alpha = -10,130 \text{ kN}$$

$$M_B = -50,654 \text{ kNm}$$

ODCINEK BC

$$N = -49,141 \text{ kN}$$

$$T = 48,637 - 10,0 \cdot x^P$$

$$x^P = 0 \rightarrow T_B = 48,637 \text{ kN}$$

$$x^P = 4,0 \rightarrow T_C^L = 8,637 \text{ kN}$$

$$M = 48,637 \cdot x^P - 50,654 - 10,0 \cdot x^P \cdot \frac{x^P}{2}$$

$$x^P = 0 \rightarrow M_B = -50,654 \text{ kNm}$$

$$x^P = 4,0 \rightarrow M_C^L = 63,892 \text{ kNm}$$

ODCINEK DC

$$N = -63,751 \text{ kN}$$

$$T = -31,363 + 10,0 \cdot x^L$$

$$x^L = 0 \rightarrow T_D = -31,363 \text{ kN}$$

$$x^L = 4,0 \rightarrow T_C^P = 8,637 \text{ kN}$$

$$M = 31,363 \cdot x^L - 10,0 \cdot x^L \cdot \frac{x^L}{2}$$

$$x^L = 0 \rightarrow M_D = 0 \text{ kNm}$$

$$x^L = 4,0 \rightarrow M_C^P = 45,452 \text{ kNm}$$

$$T = 0 \Rightarrow -31,363 + 10,0 \cdot x^L = 0$$

$$\mapsto x^L = 3,14 \text{ m}$$

$$\text{dla } x^L = 3,14 \text{ m} \rightarrow M_{MAX}$$

$$\mapsto M_{MAX} = 49,182 \text{ kNm}$$

ODCINEK EC

$$N = 0 \text{ kN}$$

$$T = 5,39 - 20,0 = -14,61 \text{ kN}$$

$$M = 5,39 \cdot y^D - 20,0 \cdot (y^D - 2,0)$$

$$y^D = 2,0 \rightarrow M_E = 10,78 \text{ kNm}$$

$$y^D = 4,0 \rightarrow M_C^D = -18,44 \text{ kNm}$$

ODCINEK FE

$$N = 0 \text{ kN}$$

$$T = 5,39 \text{ kN}$$

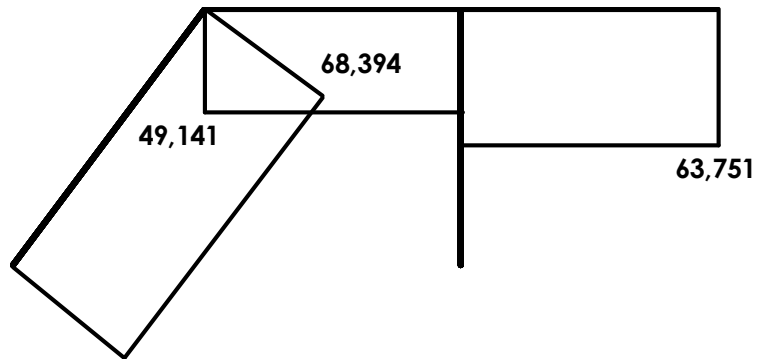
$$M = 5,39 \cdot y^D$$

$$y^D = 0 \rightarrow M_F = 0 \text{ kNm}$$

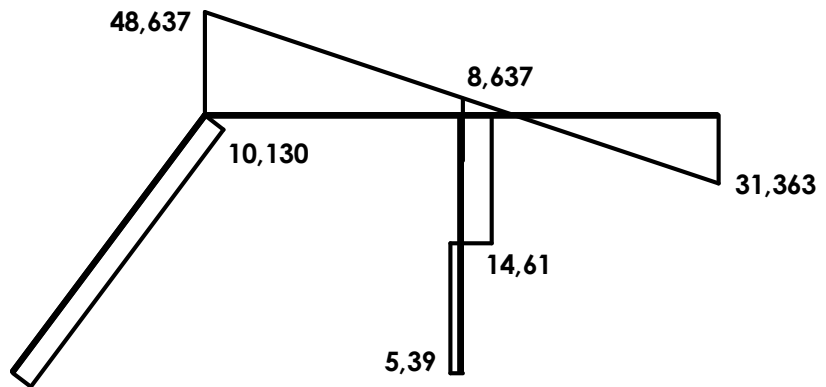
$$y^D = 2,0 \rightarrow M_E = 10,78 \text{ kNm}$$

WYKRESY SIŁ WEWNĘTRZNYCH

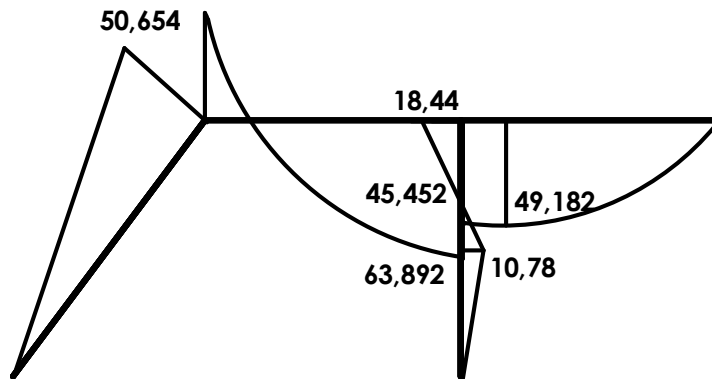
N
[kN]



T
[kN]

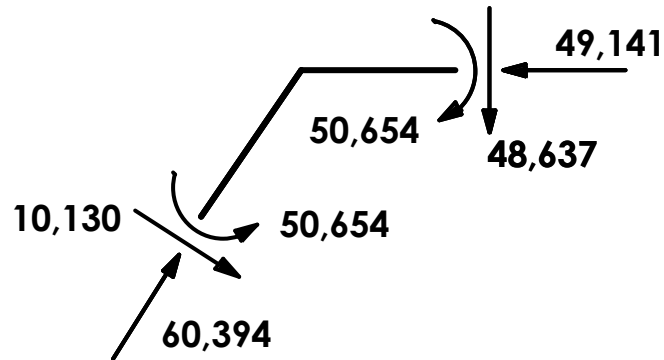


M
[kNm]



SPRAWDZENIE RÓWNOWAGI WEZŁÓW

WEZEL B

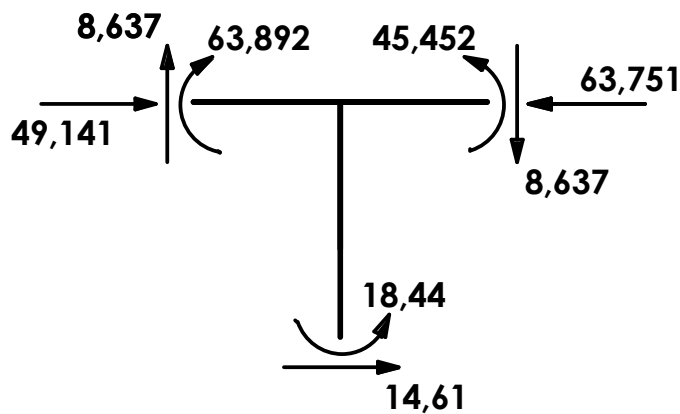


$$\sum X = 68,394 \cdot \cos \alpha + 10,130 \cdot \sin \alpha - 49,141 = -0,513 \cdot 10^{-3} \quad kN$$

$$\sum Y = 68,394 \cdot \sin \alpha - 10,130 \cdot \cos \alpha - 48,637 = -0,112 \cdot 10^{-3} \quad kN$$

$$\sum M = 58,654 - 58,654 = 0 \quad kNm$$

WEZEL C

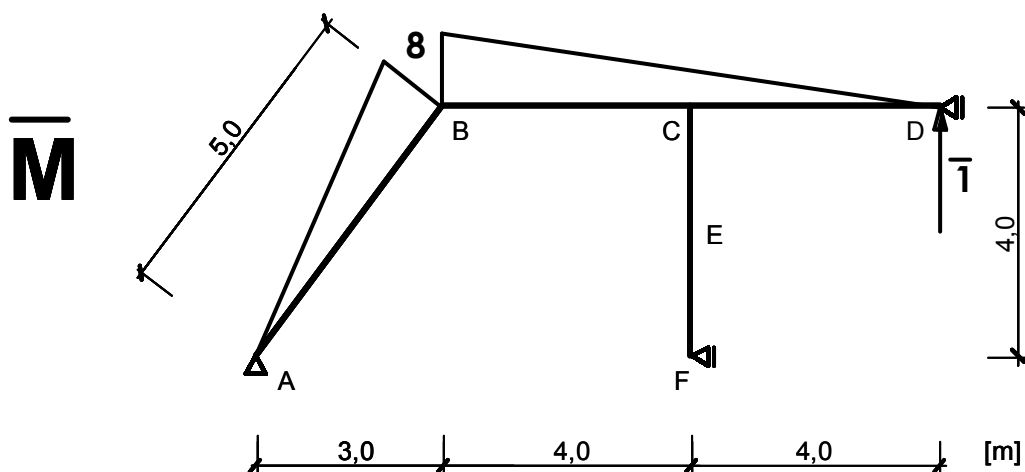
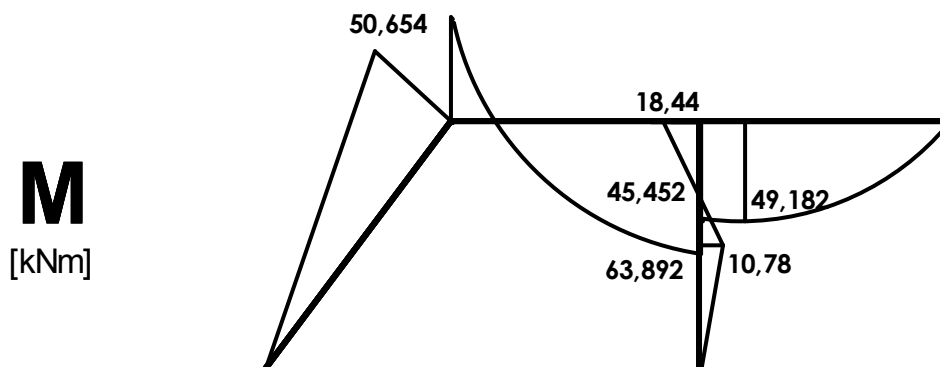


$$\sum X = 49,141 + 14,61 - 63,751 = 0 \quad kN$$

$$\sum Y = 8,637 - 8,637 = 0 \quad kN$$

$$\sum M = 63,892 - 18,44 - 45,452 = 0 \quad kNm$$

SPRAWDZENIE – KONTROLA KINEMATYCZNA
OBLICZANIE PRZEMIESZCZENIA PIONOWEGO PUNKTU D



$$\delta_D^v = \frac{1}{EI} \left[\begin{aligned} & \frac{2}{3} \cdot 4 \cdot 20 \cdot \frac{1}{2} \cdot 4 + \frac{1}{2} \cdot 4 \cdot 45,452 \cdot \frac{2}{3} \cdot 4 + \frac{2}{3} \cdot 4 \cdot 20 \cdot \frac{1}{2} \cdot (8+4) + \\ & + \frac{4}{6} \cdot (-2 \cdot 50,654 \cdot 8 + 2 \cdot 63,892 \cdot 4 - 50,654 \cdot 4 + 63,892 \cdot 8) - \frac{1}{2} \cdot 5 \cdot 50,654 \cdot \frac{2}{3} \cdot 8 \end{aligned} \right] = -\frac{0,182}{EI} \approx 0$$

KONIEC