Z.1. BUDOWA MACIERZY CHARAKTERYSTYCZNEJ

Funkcje podstawowe opisano w lokalnym układzie współrzędnych związanym z punktem źródłowym. Przyjęto oznaczenia: $r_i = \sqrt{n_i^2 + s_i^2}$, $i = 1, 2, 3... le$.

Z.1.1. Funkcje podstawowe dla płyty nieograniczonej, obciążonej siłą skupioną $P_i^* = 1^*$

$$w_i^* = \frac{1}{8\pi D} r_i^2 \ln(r_i) \tag{Z.1}$$

$$\varphi_{n_i}^* = \frac{1}{4\pi D} n_i \ln(r_i) + \frac{1}{8\pi D} n_i \tag{Z.2}$$

$$\varphi_{s_i}^* = \frac{1}{4\pi D} s_i \ln(r) + \frac{1}{8\pi D} s_i \tag{Z.3}$$

$$M_{n_i}^* = -\frac{1}{4\pi}\left[(1+\nu_\mathrm{p})\ln(r_i) + \frac{n_i^2 + \nu_\mathrm{p} s_i^2}{r_i^2} + \frac{1}{2}(1+\nu_\mathrm{p})\right] \tag{Z.4}$$

$$M_{s_i}^* = -\frac{1}{4\pi}\left[(1+\nu_\mathrm{p})\ln(r_i) + \frac{s_i^2 + \nu_\mathrm{p} n_i^2}{r_i^2} + \frac{1}{2}(1+\nu_\mathrm{p})\right] \tag{Z.5}$$

$$M_{ns_i}^* = -\frac{1-\nu_\mathrm{p}}{4\pi}\frac{n_i s_i}{r_i^2} \tag{Z.6}$$

$$T_{n_i}^* = -\frac{1}{2\pi}\frac{n_i}{r_i^2} \tag{Z.7}$$

$$T_{s_i}^* = -\frac{1}{2\pi}\frac{s_i}{r_i^2} \tag{Z.8}$$

Z.1.2. Funkcje podstawowe dla płyty nieograniczonej, obciążonej momentem skupionym $M_{n_i}^* = 1^*$

$$\overline{w}_i^* = \frac{1}{4\pi D} n_i \ln(r_i) \tag{Z.9}$$

$$\overline{\varphi}_{n_i}^* = \frac{1}{4\pi D}\left[\ln(r_i) + \frac{n_i^2}{r^2}\right] \tag{Z.10}$$

$$\overline{\varphi}_{s_i}^* = \frac{1}{4\pi D}\frac{n_i s_i}{r_i^2} \tag{Z.11}$$

$$\overline{M}^{*}_{n_i} = -\frac{1}{4\pi}\frac{n_i}{r_i^2}\left[\left(1+\nu_{\mathrm{p}}\right)+2\left(1-\nu_{\mathrm{p}}\right)\frac{s_i^2}{r_i^2}\right] \tag{Z.12}$$

$$\overline{M}^{*}_{s_i} = -\frac{1}{4\pi}\frac{n_i}{r_i^2}\left[\left(1+\nu_{\mathrm{p}}\right)-2\left(1-\nu_{\mathrm{p}}\right)\frac{s_i^2}{r_i^2}\right] \tag{Z.13}$$

$$\overline{M}^{*}_{n_i s_i} = \frac{1-\nu_{\mathrm{p}}}{4\pi}\frac{s_i}{r_i^2}\left(2\frac{n_i^2}{r_i^2}-1\right) \tag{Z.14}$$

$$\overline{T}^{*}_{n_i} = \frac{1}{2\pi}\left(\frac{n_i^2-s_i^2}{r_i^4}\right) \tag{Z.15}$$

$$\overline{T}^{*}_{s_2} = \frac{1}{\pi}\left(\frac{n_i s_i}{r_i^4}\right) \tag{Z.16}$$

Z.2. OBLICZANIE UGIĘCIA

Układ współrzędnych przesunięto równolegle i zaczepiono w punkcie, w którym obliczane jest ugięcie. Przyjęto oznaczenia: $x_1 = x$, $x_2 = y$ i $r = \sqrt{x^2 + y^2}$.

Z.2.1. Funkcje podstawowe dla płyty nieograniczonej, obciążonej siłą skupioną $P^* = 1^*$

$$w^* = \frac{1}{8\pi D} r^2 \ln(r) \tag{Z.17}$$

$$\varphi_x^* = \frac{1}{4\pi D} x \ln(r) + \frac{1}{8\pi D} x \tag{Z.18}$$

$$\varphi_y^* = \frac{1}{4\pi D} y \ln(r) + \frac{1}{8\pi D} y \tag{Z.19}$$

$$M_x^* = -\frac{1}{4\pi}\left[(1+\nu_\mathrm{p})\ln(r) + \frac{x^2 + \nu_\mathrm{p} y^2}{r^2} + \frac{1}{2}(1+\nu_\mathrm{p})\right] \tag{Z.20}$$

$$M_y^* = -\frac{1}{4\pi}\left[(1+\nu_\mathrm{p})\ln(r) + \frac{y^2 + \nu_\mathrm{p} x^2}{r^2} + \frac{1}{2}(1+\nu_\mathrm{p})\right] \tag{Z.21}$$

$$M_{xy}^* = -\frac{1-\nu_\mathrm{p}}{4\pi}\frac{xy}{r^2}, \tag{Z.22}$$

$$T_x^* = -\frac{1}{2\pi}\frac{x}{r^2} \tag{Z.23}$$

$$T_y^* = -\frac{1}{2\pi}\frac{y}{r^2} \tag{Z.24}$$

Z.3. OBLICZANIE KĄTÓW OBROTU

Układ współrzędnych przesunięto równolegle i zaczepiono w punkcie, w którym obliczany jest kąt obrotu. Przyjęto oznaczenia: $x_1 = x$, $x_2 = y$ i $r = \sqrt{x^2 + y^2}$.

Z.3.1. Obliczanie kąta obrotu φ_x

$$\frac{\partial w^*}{\partial x} = \frac{1}{4\pi D} x \ln(r) + \frac{1}{8\pi D} x \tag{Z.25}$$

$$\frac{\partial \varphi_x^*}{\partial x} = \frac{1}{8\pi D} \frac{x^2 \ln(r^2) + y^2 \ln(r^2) + 3x^2 + y^2}{r^2} \tag{Z.26}$$

$$\frac{\partial \varphi_y^*}{\partial x} = \frac{1}{4\pi D} \frac{xy}{r^2}, \tag{Z.27}$$

$$\frac{\partial M_x^*}{\partial x} = \frac{1}{4\pi} \frac{x\left(-x^2 - 3y^2 - \nu_{\mathrm{p}} x^2 + \nu_{\mathrm{p}} y^2\right)}{r^4} \tag{Z.28}$$

$$\frac{\partial M_y^*}{\partial x} = \frac{-1}{4\pi} \frac{x\left(x^2 - y^2 + \nu_{\mathrm{p}} x^2 + 3\nu_{\mathrm{p}} y^2\right)}{r^4} \tag{Z.29}$$

$$\frac{\partial M_{xy}^*}{\partial x} = \frac{1}{4\pi} \frac{y(\nu_{\mathrm{p}} - 1)\left(y^2 - x^2\right)}{r^4} \tag{Z.30}$$

$$\frac{\partial T_x^*}{\partial x} = \frac{-1}{2\pi} \frac{1}{r^2} + \frac{1}{\pi} \frac{x^2}{r^4} \tag{Z.31}$$

$$\frac{\partial T_y^*}{\partial x} = \frac{1}{\pi} \frac{xy}{r^4} \tag{Z.32}$$

Z.3.2. Obliczanie kąta obrotu φ_y

$$\frac{\partial w^*}{\partial y} = \frac{1}{4\pi D} y \ln(r) + \frac{1}{8\pi D} y \tag{Z.33}$$

$$\frac{\partial \varphi_x^*}{\partial y} = \frac{1}{4\pi D} \frac{xy}{r^2} \tag{Z.34}$$

$$\frac{\partial \varphi_y^*}{\partial y} = \frac{1}{8\pi D} \frac{x^2 \ln(r^2) + y^2 \ln(r^2) + x^2 + 3y^2}{r^2} \tag{Z.35}$$

$$\frac{\partial M_x^*}{\partial y} = \frac{-1}{4\pi} \frac{y\left(y^2 - x^2 + \nu_{\mathrm{p}} y^2 + 3\nu_{\mathrm{p}} x^2\right)}{r^4} \tag{Z.36}$$

$$\frac{\partial M_x^*}{\partial x} = \frac{1}{4\pi} \frac{y\left(-y^2 - 3x^2 + \nu_{\mathrm{p}} x^2 - \nu_{\mathrm{p}} y^2\right)}{r^4} \tag{Z.37}$$

$$\frac{\partial M_{xy}^*}{\partial x} = \frac{1}{4\pi} \frac{x\left(\nu_{\mathrm{p}} - 1\right)\left(x^2 - y^2\right)}{r^4} \tag{Z.38}$$

$$\frac{\partial T_x^*}{\partial y} = \frac{1}{\pi} \frac{xy}{r^4} \tag{Z.39}$$

$$\frac{\partial T_y^*}{\partial y} = \frac{-1}{2\pi} \frac{1}{r^2} + \frac{1}{\pi} \frac{y^2}{r^4} \tag{Z.40}$$

Z.4. OBLICZANIE MOMENTÓW ZGINAJĄCYCH I MOMENTU SKRĘCAJĄCEGO

Układ współrzędnych przesunięto równolegle i zaczepiono w punkcie, w którym obliczane są momenty zginające i moment skręcający. Przyjęto oznaczenia: $x_1 = x$, $x_2 = y$ i $r = \sqrt{x^2 + y^2}$.

Drugie pochodne funkcji podstawowych:

$$\frac{\partial^2 w^*}{\partial x^2} = \frac{1}{4\pi D}\left[\ln\left(\sqrt{x^2 + y^2}\right) + \frac{x^2}{r^2} + \frac{1}{2}\right] \tag{Z.41}$$

$$\frac{\partial^2 w^*}{\partial y^2} = \frac{1}{4\pi D}\left[\ln\left(\sqrt{x^2 + y^2}\right) + \frac{y^2}{r^2} + \frac{1}{2}\right] \tag{Z.42}$$

$$\frac{\partial^2 w^*}{\partial x \partial y} = \frac{1}{4\pi D}\frac{xy}{r^2} \tag{Z.43}$$

$$\frac{\partial^2 \varphi_x^*}{\partial x^2} = \frac{1}{2\pi D}\left[\frac{3}{2}\frac{x}{r^2} - \frac{x^3}{r^4}\right] \tag{Z.44}$$

$$\frac{\partial^2 \varphi_x^*}{\partial y^2} = \frac{1}{2\pi D}\left[\frac{1}{2}\frac{x}{r^2} - \frac{xy^2}{r^4}\right] \tag{Z.45}$$

$$\frac{\partial^2 \varphi_x^*}{\partial x \partial y} = \frac{1}{2\pi D}\left[\frac{1}{2}\frac{y}{r^2} - \frac{x^2 y}{r^4}\right] \tag{Z.46}$$

$$\frac{\partial^2 \varphi_y^*}{\partial x^2} = \frac{1}{2\pi D}\left[\frac{1}{2}\frac{y}{r^2} - \frac{x^2 y}{r^4}\right] \tag{Z.47}$$

$$\frac{\partial^2 \varphi_y^*}{\partial y^2} = \frac{1}{2\pi D}\left[\frac{3}{2}\frac{y}{r^2} - \frac{y^3}{r^4}\right] \tag{Z.48}$$

$$\frac{\partial^2 \varphi_y^*}{\partial x \partial y} = \frac{1}{2\pi D}\left[\frac{1}{2}\frac{x}{r^2} - \frac{xy^2}{r^4}\right] \tag{Z.49}$$

$$\frac{\partial^2 M_x^*}{\partial x^2} = \frac{1}{4\pi}\frac{x^4 + 6x^2y^2 - 3y^4 + \nu_p x^4 - 6\nu_p x^2 y^2 + \nu_p y^4}{r^6} \tag{Z.50}$$

$$\frac{\partial^2 M_x^*}{\partial y^2} = \frac{-1}{4\pi}\frac{6x^2y^2 - x^4 - y^4 - 6\nu_p x^2 y^2 - \nu_p y^4 + 3\nu_p x^4}{r^6} \tag{Z.51}$$

$$\frac{\partial^2 M_x^*}{\partial x \partial y} = \frac{1}{2\pi} \frac{xy\left(-x^2 + 3y^2 + 3\nu_p x^2 - \nu_p y^2\right)}{r^6} \tag{Z.52}$$

$$\frac{\partial^2 M_y^*}{\partial x^2} = \frac{1}{4\pi} \frac{x^4 - y^4 - 6x^2y^2 + \nu_p x^4 + 6\nu_p x^2 y^2 - 3\nu_p y^4}{r^6} \tag{Z.53}$$

$$\frac{\partial^2 M_y^*}{\partial y^2} = \frac{1}{4\pi} \frac{6x^2y^2 + y^4 - 3x^4 + \nu_p x^4 - 6\nu_p x^2 y^2 + \nu_p y^4}{r^6} \tag{Z.54}$$

$$\frac{\partial^2 M_x^*}{\partial x \partial y} = \frac{-1}{2\pi} \frac{xy\left(-3x^2 + y^2 - 3\nu_p y^2 + \nu_p x^2\right)}{r^6} \tag{Z.55}$$

$$\frac{\partial^2 M_{xy}^*}{\partial x^2} = \frac{3}{2\pi} \frac{\left(1-\nu_p\right)xy}{r^4} - \frac{2}{\pi} \cdot \frac{\left(1-\nu_p\right)x^3 y}{r^6} \tag{Z.56}$$

$$\frac{\partial^2 M_{xy}^*}{\partial y^2} = \frac{3}{2\pi} \frac{\left(1-\nu_p\right)xy}{r^4} - \frac{2}{\pi} \cdot \frac{\left(1-\nu_p\right)xy^3}{r^6} \tag{Z.57}$$

$$\frac{\partial^2 M_{xy}^*}{\partial x \partial y} = \frac{-1}{4\pi} \frac{\left(\nu_p - 1\right)\left(x^4 - 6x^2y^2 + y^4\right)}{r^6} \tag{Z.58}$$

$$\frac{\partial^2 T_x^*}{\partial x^2} = \frac{3}{\pi} \frac{x}{r^4} - \frac{4}{\pi} \frac{x^3}{r^6} \tag{Z.59}$$

$$\frac{\partial^2 T_x^*}{\partial x^2} = \frac{1}{\pi} \frac{x}{r^4} - \frac{4}{\pi} \frac{xy^2}{r^6} \tag{Z.60}$$

$$\frac{\partial^2 T_x^*}{\partial x \partial y} = \frac{1}{\pi} \frac{y}{r^4} - \frac{4}{\pi} \frac{x^2 y}{r^6} \tag{Z.61}$$

$$\frac{\partial^2 T_y^*}{\partial x^2} = \frac{1}{\pi} \frac{y}{r^4} - \frac{4}{\pi} \frac{x^2 y}{r^6} \tag{Z.62}$$

$$\frac{\partial^2 T_y^*}{\partial y^2} = \frac{3}{\pi} \frac{y}{r^4} - \frac{4}{\pi} \frac{y^3}{r^6} \tag{Z.63}$$

$$\frac{\partial^2 T_x^*}{\partial x \partial y} = \frac{1}{\pi} \frac{x}{r^4} - \frac{4}{\pi} \frac{xy^2}{r^6} \tag{Z.64}$$

Z.5. OBLICZANIE ELEMENTÓW MACIERZY D

Przyjęto oznaczenia:

$$x_1 = x\,,\quad x_2 = y\,,\quad r = \sqrt{x^2 + y^2}\,,$$

$$x_p = x_m - \frac{a_x}{2}\,,\quad x_k = x_m + \frac{a_x}{2}\,,\quad y_p = y_m - \frac{a_y}{2}\,,\quad y_k = y_m + \frac{a_y}{2}\,,$$

$$r_1 = \sqrt{x_p^2 + y_p^2}\,,\quad r_2 = \sqrt{x_k^2 + y_p^2}\,,\quad r_3 = \sqrt{x_k^2 + y_k^2}\,,\quad r_4 = \sqrt{x_p^2 + y_k^2}\,,$$

$$\int_{\Omega_0} \frac{1}{r} \cdot d\Omega_0 = x_p \ln\left(\frac{y_p + r_1}{y_k + r_4}\right) - y_k \ln\left(\frac{x_p + r_4}{x_k + r_3}\right) + x_k \ln\left(\frac{y_k + r_3}{y_p + r_2}\right) - y_p \ln\left(\frac{x_k + r_2}{x_p + r_1}\right) \tag{Z.65}$$